INSTRUCTION IN MATHEMATICS

BULLETIN, 1932, No. 17

MONOGRAPH No. 23

UNITED STATES DEPARTMENT OF THE INTERIOR
OFFICE OF EDUCATION

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UNITED STATES DEPARTMENT OF THE INTERIOR
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COMMISSIONER

INSTRUCTION IN MATHEMATICS

BY

EDWIN S. LIDE

BULLETIN 1932, NO. 17

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NOTE

Edwin S. Lide, the author of this monograph, is specialist in curriculum of the NATIONAL SURVEY OF SECONDARY EDUCATION. *William John Cooper, United States commissioner of education, is director of the Survey; Leonard V. Koos, professor of secondary education at the University of Chicago, is associate director; and Carl A. Jessen, specialist in secondary education of the Office of Education, is coordinator.*

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LETTER OF TRANSMITTAL

DEPARTMENT OF THE INTERIOR,
OFFICE OF EDUCATION,
Washington, D. C., January, 1933.

SIR: Within a period of 30 years the high-school enrollment has increased from a little over 10 per cent of the population of high-school age to more than 50 per cent of that population. This enrollment is so unusual for a secondary school that it has attracted the attention of Europe, where only 8 to 10 per cent attend secondary schools. Many European educators have said that we are educating too many people. I believe, however, that the people of the United States are now getting a new conception of education. They are coming to look upon education as a preparation for citizenship and for daily life rather than for the money return which comes from it. They are looking upon the high school as a place for their boys and girls to profit at a period when they are not yet acceptable to industry.

In order that we may know where we stand in secondary education, the membership of the North Central Association of Colleges and Secondary Schools four years ago took the lead in urging a study. It seemed to them that it was wise for such a study to be made by the Government of the United States rather than by a private foundation; for if such an agency studied secondary education, it might be accused either rightly or wrongly of a bias toward a special interest. When the members of a committee of this association appeared before the Bureau of the Budget in 1928, they received a very courteous hearing. It was impossible, so the Chief of the Budget Bureau thought, to obtain all the money which the commission felt desirable; with the money which was obtained, $225,000, to be expended over a 3-year period, it was found impossible to do all the things that the committee had in mind. It was possible, however, to study those things which pertained strictly to secondary education, that is, its organization; its curriculum, including some of the more fundamental subjects, and particularly those subjects on which a comparison could be made between the present and earlier periods; its extracurriculum, which is almost entirely new in the past 30 years; the pupil population; and administrative and supervisory problems, personnel, and activities.

The handling of this survey was intrusted to Dr. Leonard V. Koos, of the University of Chicago. With great skill he has, working on a full-time basis during his free quarters from the University of Chicago and part time during other quarters, brought it to a conclusion.

This manuscript on mathematics was prepared by Dr. Edwin S. Lide, one of the full-time workers on the Survey. It gives attention to courses of study mostly in mimeographed form which he succeeded in obtaining and to personal visitations which he made to the various schools. It covers the work in the junior high school and in the senior high school. The wide effect of the National Committee on Mathematics on the courses of study and on practice are here recorded.

The objectives given in the junior high school are, first of all, the practical ones. These are followed in minor degree by disciplinary and cultural aims. The course itself is in 60 per cent of the schools designated as either general mathematics or simply as mathematics. Arithmetic, however, is the course yet outlined for approximately 40 per cent of the schools. Mathematics is required in grade 7, usually required in grade 8, and required also in about half of the schools in grade 9. When offered the course in general mathematics is usually recommended as a required course in all three of these grades. In the senior high school Doctor Lide found less change due to the fact that these courses more nearly are college preparatory. Geometry is the course usually offered in grade 10 although there are some variations from this practice. Work in grades 11 and 12 is still chiefly college preparatory. The results obtained in attempting to fuse algebra and analytic geometry and differential calculus with advanced geometry and trigonometry have not met with much success.

In most cases the outlines of the courses of study are closely connected with the textbook. In Boston, teachers appeared less dependent on the textbook than elsewhere. Improved methods in selecting textbooks are in use in some of the cities, notably, Los Angeles. The work of grades 9 to 12 seems to be considerably influenced by college-entrance examinations, especially in the East; the West appears to be less under the influence of these.

The manuscript is an interesting presentation of mathematics as it exists in the country at the present time, and I respectfully recommend that it be printed as one of the monographs in the National Survey of Secondary Education.

Respectfully submitted.

WM. JOHN COOPER,
Commissioner.

THE SECRETARY OF THE INTERIOR.

INSTRUCTION IN MATHEMATICS

CHAPTER I : GENERAL CHARACTERISTICS OF OUTLINES OF COURSES

1. NATURE OF THE STUDY

Manner of securing data.—Plans for the treatment of mathematics instruction in this survey have proceeded on the theory that more interest and value will result from a presentation of outstanding practices and innovations in this field than from a mere survey of status. In accord with this theory, general inquiries went to each State department of education and to several thousand city superintendents and high-school principals requesting that they indicate the lines along which they were doing outstanding work and that they send outlines of their recent courses of study. To the outlines secured in this manner were added a number loaned by the research division of the National Education Association. All such materials were carefully analyzed.

As a result of these procedures, together with an analysis of literature applicable and correspondence with leaders in the field requesting that they send names of schools outstanding in mathematics instruction which were known to them, a list of schools for visitation was also compiled. One to two-day visits were made in each of the schools selected.

General nature of report.—This report presents practices, therefore, in what may be considered a selected group of schools in the United States. Although analysis was made of all outlines submitted, the fact that a school publishes its course of study and releases it for analysis is indicative of efforts to produce a better course of study than obtains in the average school. The schools visited were chosen because their work in mathematics was considered outstanding.

The first chapter considers general characteristics revealed from an analysis of the outlines only. In the second and third chapters a more detailed presentation is made of

practices in grades 7, 8, and 9 and in grades 10, 11, and 12; that is, in what may be termed, respectively, the grades of the junior and of the senior high school. It is not that all schools considered are organized in this manner, but that such designation is now rather general. The fourth chapter is devoted to influences affecting the production and use of courses of study, and the final chapter to a general summary.

Source of outlines examined.—The courses of study analyzed represent 79 schools and systems scattered over the United States. The distribution of these cities by region and by population groups is shown in Table 1.

TABLE 1.—*Distribution of city courses of study in mathematics*

Region	Number of cities by population groups					
	Fewer than 10,000	10,000 to 30,000	30,000 to 100,000	100,000 to 500,000	More than 500,000	Total
1	2	3	4	5	6	7
New England			2	4	1	7
Mid-Atlantic	1	2	9	3	4	19
Southern	1	1	2	3		7
Mid-West	4	3	12	7	5	31
West	3	1	3	6	2	15
Total	9	7	28	23	12	79

Form in which outlines were published.—Although only 79 cities are represented, 103 separate courses of study in mathematics which were published by them are included in the analysis. Classification as to form in which published and number of pages included are shown in Table 2. In some

TABLE 2.—*Method of assembling and number of pages included in the different city courses of study*

Method of assembling	Fewer than 10 pages	10 to 50 pages	50 to 100 pages	More than 100 pages	Total
1	2	3	4	5	6
Printed and bound	2	9	10	6	27
Mimeographed and bound	7	24	8	4	43
Mimeographed and loose leaf	6	19			25
Typed and loose leaf	4	1			5
Other	1	2			3
Total	20	55	18	10	103

cities more than one form is followed in that those courses which are considered as tentative are mimeographed and loose leaf, while those considered as more or less final for a few years are printed.

No outlines are represented if published earlier than 1925, and only 16 per cent of the total are dated earlier than 1928. Forty-six per cent of the 103 courses are dated 1928, 1929, or 1930; 3 per cent are dated 1931, while 35 per cent, although bearing no published date, fall within the 6-year period included. It seems safe to say, therefore, that the majority of the outlines examined were published in 1928, 1929, or 1930 and represent the latest productions of the schools considered.

1. CHARACTERISTICS OF LOCAL OUTLINES

Variations in content.—One of the first impressions gained from an examination of the outlines is the great variation in the manner of treating the major phases of mathematics instruction. At one extreme a course of study may contain a bare list of objectives, an informal account of the general procedures for teaching, and an organized outline of the materials to be taught in a specified grade or course. There may be no suggestions concerning the measurement of the results of instruction, the treatment of individual differences, or additional reference materials which might aid the teacher in her classroom work. At the other extreme, however, may be found a course of study arranged in three parallel columns containing (1) a carefully worked out list of objectives and outcomes; (2) opposite each objective, pupil activities and reading materials arranged for groups classified according to ability; (3) carefully outlined procedures, references to additional procedures, methods of testing results, methods of treating individual differences, and the like.

In courses recently constructed by systems in which revision has been a policy over a period of years there seems to be a tendency, first, to orient the teacher regarding the whole course, and, second, to follow with specific details of procedure and method applicable to the various topics included in each semester's work. For example, in the 1931 course constructed at Denver the first part of the outline is devoted

to a discussion of objectives, the arrangement of the content, general methods and suggestions, tests, and a chart for the semester outlining the work for superior, medium, and limited groups. A chapter is then devoted to each of the main units of content in which methods and activities for developing that unit are treated.

3. GENERAL PHASES OF INSTRUCTION TREATED

Manner of treating seven phases.—The manner in which the different committees treated certain major phases of instruction in the courses of study represented in this report is shown in Tables 3 and 4.[1] The practices in 57 centers in which the outlines were bound separately for the junior high school alone or for junior and senior high school together are shown in Table 3, while in Table 4 are shown the practices in 22 centers in which outlines for senior or 4-year high-school courses were bound separately. It will be noted that in practically all outlines some attention has been given to objectives, teaching procedures, and content materials, but that in this selected group of schools, tests, individual differences, references, and miscellaneous phases have received attention in less than half. Miscellaneous phases includes such items as methods used in the process of revision, inventory of previous pupil attainments, and the like.

TABLE 3.—*Manner of treating the major phases of junior or junior-senior high school courses*

Phase	Informal account	Bare list	Organized outline	Parallel columns	Division into units	Organized paragraph	Not treated
1	2	3	4	5	6	7	8
Objectives	5	32	4	6	.....	1	9
Procedures	16	7	5	10	.....	7	12
Content	5	16	20	10	5	.....	1
Tests	5	10	7	.....	1	.....	34
Individual differences	7	2	1	4	.....	1	42
References	.....	21	2	2	.....	.....	32
Miscellaneous	15	4	1	.....	.....	.....	37

[1] The headings listed for the manner of treating courses are an adaptation of those followed by Henry Harap in A Critique of Public-School Courses of Study, 1928-29. Journal of Educational Research, 24: 109-119, February, 1930.

TABLE 4.—*Manner of treating the major phases of senior or 4-year high school courses*

Phase	Informal account	Bare list	Organized outline	Parallel columns	Division into units	Organized paragraph	Not treated
1	2	3	4	5	6	7	8
Objectives	1	13	2			1	5
Procedures	7	3	2		2	2	6
Content	1	9	7	1	4		
Tests	3	1	2				16
Individual differences	4		1	1		1	15
References		8	1				13
Miscellaneous	7	1	2				12

In treating objectives a slight majority in each type of school merely gives a bare list. Teaching procedures are treated in the greater number of cases through informal account, but there is a much wider distribution. The method of treating content materials is fairly well divided between those schools in which only a bare list is given and others in which an organized outline is devoted to them. In a fairly large group of schools also, both content and teaching procedures are treated in parallel columns. By this method the printed page is divided into parallel columns, which, for example, may be headed "Objectives," "Procedures," and "Content." Opposite each objective appears in the proper column the procedures and subject-matter materials for its realization. In the case of tests, only a bare list is generally given; individual differences are treated through informal account; references, by tabulating a bare list; and miscellaneous phases through informal account. In a few cases organized paragraphs are devoted to a discussion of teaching procedures and in a few others content materials are organized into units of instruction. Otherwise these two methods are hardly used.

Altogether, the large number of cases in which phases are treated by merely giving a bare list creates the impression that the treatment of outlines is of a formal nature. It is evidently to guard against a consequent reaction in teaching that the following statement appears in the foreword of one of the outlines: "While the outlines are topical, and hence formal, it is hoped that teachers will be able to teach by

natural life activities embracing in the same day more than one topic, and that they will use the formal outline mainly as a check upon the ground covered."

Percentage of space devoted to each phase.—Wider disagreement is evidenced in the proportion of space devoted to each of the major phases under discussion. Data are tabulated in Figure 1, showing for the entering grade (7 and 10) under the junior and under the senior high school organization the percentage of space devoted in the outlines to the major phases of instruction by the median and the range of the middle 50 per cent of schools.

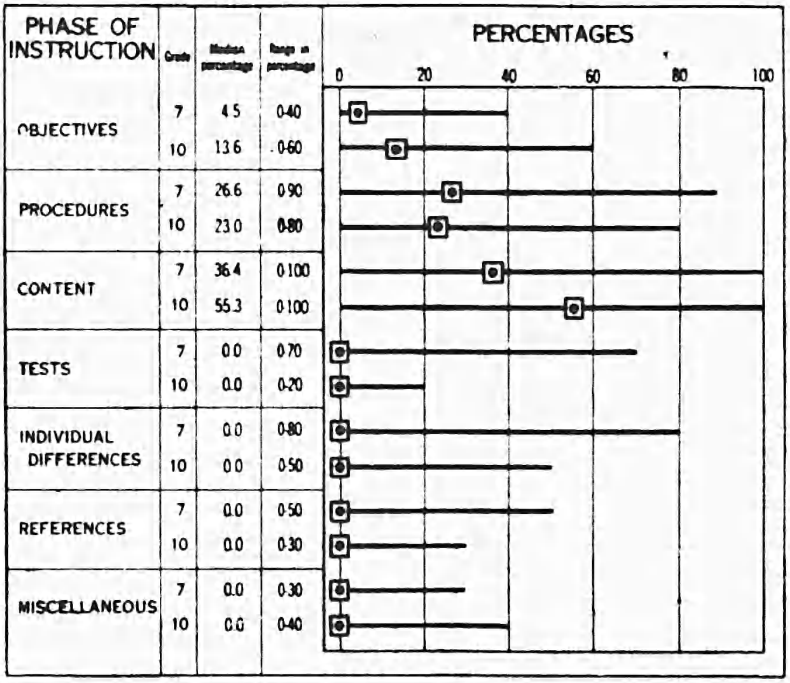

FIGURE 1.—The percentages of space devoted to certain major phases of instruction in courses of study for mathematics. (Median and range of the middle 50 per cent of schools shown for grades 7 and 10)

In both grades and for all phases, except for procedures and content, the proportion of space seldom runs over 30 per cent. In the case of these two, however, the variation is from 0 to 100 per cent. Subject-matter materials, followed rather closely by teaching procedures, claim the largest proportion of space in almost every outline. The principal distinction between grade 7 and grade 10 is in the greater pro-

portion of space devoted to content in the latter. This is, for the most part, at the expense of tests, individual differences and references, to which somewhat more attention is devoted in the outlines for grade 7. Junior high school outlines, in other words, appear to be less formal and more suggestive than those from senior high school.

4. SPECIFIC CHARACTERISTICS OF CONTENT AND FORM

Six major items considered.—Some cause of the wide disagreement in the proportion of space devoted to the separate phases noted in an earlier paragraph is shown in Table 5, in which certain characteristics of the content and form of the various outlines are indicated under objectives, selection and organization of materials of instruction, individual differences, teaching procedures, testing, and mechanical phases.

TABLE 5.—*Number of local course outlines which exhibit certain characteristics in content and form*

Characteristic	Number of outlines	
	Junior high school (57)	Senior high school (46)
I. OBJECTIVES		
1. All secondary education	9	6
2. Junior high school or senior high school	6	
3. Secondary mathematics	8	20
4. Junior or senior high school mathematics	32	7
5. Grade or course	28	42
6. Specific outcomes for each grade or course	24	10
7. Broad aims related to major objectives	8	3
8. Detailed aims related to broad aims	8	
9. Influence of authoritative formulations	34	31
10. Influence of local conditions	7	1
II. SELECTION AND ORGANIZATION OF MATERIALS		
1. Materials specifically related to broad aims	11	1
2. Materials specifically related to detailed aims	28	10
3. Materials specifically related to future life needs	47	6
4. Materials specifically related to children's needs	44	3
5. Selection of local materials	26	5
6. Materials organized psychologically	27	7
7. Materials organized logically	20	39
8. Some materials chosen from other subject fields	52	6
9. Materials organized on unit plan (other than textbook)	13	7
10. Relative emphasis on topics within course	33	19
11. Specific materials for reteaching, relearning	24	1
12. Suggestions for articulation	14	6
13. Time allotments	35	25
14. Indication of use of objective studies	9	1

TABLE 5.—*Number of local course outlines which exhibit certain characteristics in content and form*—Continued

Characteristic	Number of outlines	
	Junior high school (57)	Senior high school (46)
III. INDIVIDUAL DIFFERENCES		
1. Specific materials selected	19	13
2. Materials for ability groups	9	10
3. Suggestions as to use of laboratory plan	9	7
4. Suggestions as to directed study	6	5
5. Suggestive supplementary activities familiar to pupil	26	12
6. Activities distinct for knowledges, habits, attitudes, etc	14	1
7. Connection of activities to mastery of specific objectives	27	8
8. Procedures suggestive for individual progress	24	9
9. Suggestions as to experimentation	4	3
10. Indication of use of objective studies	10	.....
IV. TEACHING PROCEDURES		
1. General procedures applicable to entire course	31	34
2. Procedures connected with outline of specific materials	36	22
3. Illustrative and type lessons	5	6
4. Suggestions as to topics of local interest	38	10
5. Suggestions as to correlation with other subject fields	9	7
6. Related to basic textual materials	29	22
7. Attention to attractive style of writing	7	3
8. Suggestions as to pupil use of study materials	18	8
9. Suggestions as to visual aids	12	8
10. Suggestions as to corrective and practice materials	30	9
11. References to additional subject matter	28	20
12. References to methods and theory of teaching	26	18
V. MEASURING THE LEARNING PRODUCT		
1. General standards for classification and promotion	18	5
2. Detailed statement of expected outcomes of learning	19	9
3. Suggestions for testing knowledges and skills	35	13
4. Suggestions for testing attitudes and appreciations	7	.....
5. Use of objective studies in determining standards	6	.....
VI. MECHANICAL MAKE-UP		
1. Attention to convenience of reference	22	8
2. Provision for teacher revision	6	3
3. Mechanical aids to emphasis	15	3

NOTE.—The numbers in parentheses indicate the number of outlines represented.

Objectives in junior high school.—Under the heading "Objectives," the first six items in the table refer to types of objectives which may be listed in a junior high school outline, while the last four of the ten refer to relationships or influences on the formulation. Only those designated as "Junior or senior high school mathematics" were found in as many as half the total number. Objectives for a specific grade or course and specific outcomes for a grade or course are listed in almost half of the 57 outlines, but objectives

for secondary education, for the junior high school, and for all mathematics in the secondary school are listed only rarely. Only eight schools are shown here in which the relationship between the different types of aims is brought out in the course outline. As is shown under the second major heading of the table, however, the number of outlines in which materials of instruction are related to aims is large.

Statement of objectives in 34 of the 57 outlines reflect the influence of authoritative formulations. In all but one or two cases the list of objectives drawn up by the National Committee on the Reorganization of Mathematics as published in 1923 is reflected. In some cases the exact list of this committee was adopted for the local system. While the literature of seventh and eighth grade arithmetic emphasizes teaching that has practical application, in only seven outlines did the statement of objectives reflect the influence of local conditions.

Objectives in senior high school.—In the senior high school outlines the proportions just noted are markedly less in all cases except in the number of schools stating objectives for secondary mathematics, objectives for the grade or course, and in the number showing the influence of authoritative formulations. Judging from this, more attention is devoted to objectives in junior than in senior high school outlines.

Selection and organization of materials in junior high school.—Under selection and organization of materials, such materials are tabulated as being related to the broad aims of secondary education in only 11 junior high school outlines and to the detailed aims in scarcely half. The controversy between advocates of education as living here and now and education as preparation for the future, however, may be reflected in the fact that, in 47 of the 57 junior high school outlines, materials are specifically related to the future life needs of the pupil, and in 44 they are related to children's present needs and interests. Local materials, however, were included in scarcely half the outlines examined.

Where a consistent attempt was made throughout the outline to organize the materials of instruction into topics which are more in terms of the concepts of life than in terms of traditional topics, the organization was credited as psycholog-

ical rather than logical. Twenty-seven junior high school outlines were judged to fall entirely within the former category, as against 20 which were considered as logical. In the case of 10 outlines, neither type was thought to predominate over the other. In many cases, however, the organization of the textbook was followed in the outline. There were only 13 outlines in which individual differences were recognized through organization of materials on some one of the commonly designated "unit" plans.

The emphasis on social uses of arithmetic is reflected in the fact that of the 52 outlines considered as embodying materials from subject fields in addition to mathematics, such materials are in most cases from the social studies. In a few instances, however, were found suggestions for the inclusion of materials from art, science, English, and vocations. The social uses of mathematics are emphasized distinctly by the arrangement of the Cleveland (Ohio) outline. In this outline three objectives are listed for each semester, one of which is the use of mathematical concepts in social situations. All materials of instruction and teaching procedures contributing to this objective are listed under it.

Suggestive time allotments are also to be found in a majority of the courses. The other items listed under selection and organization of materials—i. e., specific materials for reteaching or relearning, suggestions as to articulation with the work in grades above and below, and indication as to the use of objective studies—were found in less than one-half of all outlines.

The attention given to articulating junior high school mathematics with the grades above and below as provided in the outlines at Pittsburgh, Pa., Kansas City, Mo., and Ithaca, N. Y., is worthy of mention. In these outlines an inventory of the supposedly previous mathematical attainments of the pupil is presented at the beginning of each semester's work. The outline of Ithaca also has a short summary of the attainments of the pupil in the first six grades.

Selecting and organizing materials in senior high school.—Turning now to senior high school outlines, as compared with junior high school the frequency of items listed under

selection and organization of materials is less than was shown in the case of objectives. Time allotments is the only item under selection and organization of materials which is contained in a majority of senior high school courses. In a great majority of courses the outlines follow the logical rather than the psychological method. Ten of the 14 items listed in the table under selection and organization of materials were included in fewer than 10 senior high school outlines.

Individual differences in junior high school.—Under individual differences the table shows that only one item—that of suggesting supplementary activities familiar to the pupil—is included in a majority of the junior high school outlines. Activities are shown as connected with the mastery of specific objectives in 27 outlines, and there was indication of procedures suggestive for individual progress in 24. None of the following items, however, was considered as being included in as many as 20 outlines: Selection of specific materials for individual differences or ability groups; suggestions concerning the use of directed study or a laboratory plan; the listing of activities separately for outcomes distinguished as knowledges, skills, habits or attitudes, and appreciations; suggestions lending encouragement to teacher experimentation; or indication of the use of objective studies.

Individual differences in senior high school.—Contrasting like data for the senior high school outlines with those just presented for the junior high school, again in practically all cases a decrease for the former is to be noted. The decrease is especially marked in the number of outlines in which are included suggestive supplementary activities familiar to the pupil; activities distinct for knowledges, habits, and attitudes; connection of activities to the mastery of specific objectives; procedures suggestive for individual progress; and indication of the use of objective studies.

Teaching procedures in junior high school.—In view of the amount of space devoted to teaching procedures, the frequency of appearance of the items listed under this heading is not surprising. The table shows the following to be listed in the majority of courses: The discussion of general procedures which are applicable to the whole course, the con-

nection of procedures with the outline of content materials, suggestive procedures for dealing with topics of local interest, the relation of the outline to materials in the local textbook, and suggestions concerning the use of corrective and practice materials. Suggestions as to topics of local interest are of more frequent occurrence than are actual materials of local interest. A bibliography containing references to additional materials on subject matter or methods of teaching was contained in almost half of the total number of outlines. Such items as the following, however, did not appear in as many as 20 of the outlines: Illustrative and type lessons, suggestions as to correlation with other subject fields, attractiveness in styles of writing, suggestions as to pupil use of study materials, and suggestions concerning the use of visual aids.

Teaching procedures in senior high school.—While less marked than in the first three phases, the senior high school outlines once more fall short of the record just indicated for junior high school. This is true for all items in the list except the two listed as general procedures applicable to the entire course, and illustrative and type lessons. The decrease is especially notable in the number of senior high school courses listing suggestions as to topics of local interest and suggestions as to corrective and practice materials.

Measuring the learning product in junior high school.—In measuring the results of teaching in junior high schools, almost two-thirds of the outlines had something to say regarding the testing of knowledges and skills, but only seven were found in which the testing of attitudes and appreciations was considered. General standards for classification and promotion and detailed statements of standards in the form of outcomes of instruction were found in 18 and 19 outlines, respectively, while the use of objective studies in determining standards was found in only 6.

Among junior high school outlines which devoted careful attention to the testing of knowledges and skills may be mentioned those at Cleveland, Detroit, Rochester, Springfield (Mass.), and Wichita. In some cases suggestive tests are placed at the end of each unit of content.

Measuring the learning product in senior high school.—In keeping with the tendency already noted, the senior high

school courses give attention to measuring the learning product less often than do junior high school outlines. The item listed in the greatest number of outlines is the suggestion for testing knowledges and skills.

Mechanical make-up in junior high school.—The number of course outlines which are credited as including the items listed under "Mechanical make-up" seems to indicate a lack of regard for this phase on the part of revision committees. For junior high schools, under the item, "Attention to convenience of reference," only 22 outlines are listed as having a table of contents, index, or other convenient arrangement to enable one to turn readily to an item in which one is interested. While in many instances the foreword or introduction to a course of study indicated that it was to be regarded as tentative, in only six were to be found directions to teachers for making notes on its revision. For the most part, the 15 outlines credited opposite "Mechanical aids to emphasis" are those in which the materials are arranged in parallel columns.

Mechanical make-up in senior high school.—In senior high school, attention to convenience of reference is to be noted in 8 of the 44 outlines, while the other 2 items listed in the table were each noted in only 3. The comparisons with junior high school outlines have indicated very clearly that, for practically all items, less attention was given in senior high school outlines. It seems that less care is given to the construction of senior high school outlines.

5. CHARACTERISTICS OF STATE OUTLINES

Comparison with local outlines.—Courses of study issued by State departments of education and representing for the same period of time junior high schools in 6 States and senior high schools in 10 States were also collected. In order to indicate variations, if any, from practices in local schools, the same items of information as gathered from State courses of study in mathematics are presented in Table 6. While the items represented in a majority of States for junior high school courses are also represented in a majority of local courses, on the whole it seems that all items are treated less often in State than in local courses

TABLE 6.—*Number of State courses of study which exhibit certain indicated characteristics in content and form*

Characteristic	Number of outlines	
	Junior high school (6)	Senior high school (10)
I. OBJECTIVES		
1. All secondary education		1
2. Junior high school or senior high school		
3. Secondary mathematics		7
4. Junior or senior high school mathematics	5	
5. Grade or course		5
6. Specific outcomes for each grade or course	2	5
7. Broad aims related to major objectives	1	
8. Detailed aims related to broad aims	1	
9. Influence of authoritative formulations	5	7
10. Influence of local conditions		
II. SELECTION AND ORGANIZATION OF MATERIALS		
1. Materials specifically related to broad aims	2	
2. Materials specifically related to detailed aims	2	4
3. Materials specifically related to future life needs	6	4
4. Materials specifically related to children's needs	6	3
5. Selection of local materials	1	1
6. Materials organized psychologically	6	2
7. Materials organized logically		9
8. Some materials chosen from other subject fields	6	2
9. Material organized on unit plan (other than textbook)		1
10. Relative emphasis on topics within course	5	5
11. Specific materials for reteaching, relearning		1
12. Suggestions as to articulation	3	6
13. Time allotments	1	4
14. Indication of use of objective studies	1	
III. INDIVIDUAL DIFFERENCES		
1. Specific materials selected		2
2. Materials for ability groups		1
3. Suggestions as to use of laboratory plan		1
4. Suggestions as to directed study	3	2
5. Suggestive supplementary activities familiar to pupil	6	6
6. Activities distinct for knowledges, habits, attitudes, etc	1	
7. Connection of activities to mastery of specific objectives	4	3
8. Procedures suggestive for individual progress	2	2
9. Suggestions as to experimentation		2
10. Indication of use of objective studies		
IV. TEACHING PROCEDURES		
1. General procedures applicable to entire course	4	10
2. Procedures connected with outline of specific materials	6	7
3. Illustrative and type lessons	3	3
4. Suggestions as to topics of local interest	5	5
5. Suggestions as to correlation with other subjects	1	1
6. Related to basic textual materials	1	5
7. Attention to attractive style of writing	1	1
8. Suggestions as to pupil use of study materials	2	3
9. Suggestions as to visual aids	4	5
10. Suggestions as to corrective and practice materials	2	3
11. References to additional subject matter	2	6
12. References to methods and theory of teaching	5	9
V. MEASURING THE LEARNING PRODUCT		
1. General standards for classification and promotion		2
2. Detailed statement of expected outcomes of learning	2	4
3. Suggestions for testing knowledges and skills	3	6
4. Suggestions for testing attitudes and appreciations	1	
5. Use of objective studies in determining standards		
VI. MECHANICAL MAKE-UP		
1. Attention to convenience of reference	2	2
2. Provision for teacher revision		
3. Mechanical aids to emphasis	1	2

NOTE.—The numbers in parentheses indicate the number of outlines represented.

With respect to junior high school objectives, the State outlines agree with local outlines in that a majority list objectives for junior high school mathematics and, similarly a majority show the influence of authoritative formulations. The other eight items included under objectives appear in a less proportion of State courses than they do in local junior high school courses. Likewise with regard to the remaining major topics in the table it may be said in general that they are included less frequently in State courses than in local junior high school courses.

Half the senior high school outlines list objectives for each grade or course and a majority show the influence of authoritative formulations. A majority of local senior high school courses also include these same items, but they do not include objectives for secondary mathematics shown for State courses. More attention seems to be given to the construction of senior high school courses by State authorities than by those in local systems.

6. ITEMS INCLUDED IN A COMPLETE TREATMENT

Relative emphasis.—The preceding analysis may not be interpreted as an indication of the degree of emphasis given certain items in the classrooms of the schools represented. No doubt there are schools in which each of the major topics, although they may not be represented at all in the outlines, are carefully developed. It may well be the purpose of those preparing the courses to devote major emphasis in the outline only to those topics for which special need is felt. The analysis does indicate, however, wide variation in the items which the different course-of-study committees have included; and it indicates that much more care appears to have been given in junior than in senior high school outlines to phases of instruction over which teachers may well be concerned.

Objectives.—What, then, are the items which a complete treatment of course outlines would include? For objectives, just as for each of the other major headings listed in the table, weaknesses in the local system should determine the items to which major emphasis is given. Certainly, however, each outline should include objectives for junior or senior high

school mathematics, depending on the division treated, and objectives for each grade or course. Such objectives should be set-up only after consideration of authoritative formulations in that field. Furthermore, although necessity for economy may prevent their inclusion in the outline, the objectives named should be adopted only after formulation of and articulation with objectives of all secondary education, of junior and senior high school education, and of secondary mathematics. The objectives of the first tentative draft may not include—depending on the professional training and character of experience of the group—specific objectives or outcomes for each grade or course. The ultimate aim of the committee, however, should be a detailed list of outcomes, specific, and, as far as possible, measureable, and showing careful study and influence of local conditions.

The selection and organization of materials.—The content should give evidence of being more than a logical outline of the basic textual materials. In the first place, the selection should relate specifically to the broad and to the detailed aims or objectives which have been adopted and should reflect children's present-day as well as their future life needs. In the second place, the organization should be psychological, in that it indicates serious efforts to organize the material so that it will be within the grasp of the child and, as far as possible, appeal to his interests. Such organization will involve, perhaps, some form of the "unit" plan. It will certainly involve the selection of local materials and materials from other subject fields. In the third place, there should be included materials to provide for reteaching and relearning, a practice which the senior high school outlines, especially, indicate to be infrequent. Articulation, time allotments, and indication of topics which are most to be emphasized should likewise have a place in the complete outline, and there should be indication of the use of objective studies in the determination of all items.

Individual differences.—The provision which is made for individual differences will perhaps reflect, more than any other provision, the carefulness of attention given to course-of-study construction. Certainly there should be specific materials included for this purpose, as well as procedures

suggestive for individual progress, such as the use of directed study or the laboratory plan of instruction. The ultimate aim of the committee will be the suggestion of a variety of activities, many of which have been carefully tested, which will aid in the attainments of specific types of learning, such as those for obtaining knowledge, habits, and attitudes. Again, objective studies should be the basis, as far as possible, of the materials and procedures suggested.

Teaching procedures.—The complete outline should give attention to general teaching procedures and to procedures connected with specific materials or specific types of learning. There should also be references to published articles which treat of procedures and suggest additional subject matter. The specific procedures may well treat of such matters as pupil use of study materials, visual aids, use of corrective and practice materials, topics of local interest, correlation with other subject fields, and illustrative or type lessons of a suggestive nature. More attention should be given to the presentation of course-of-study materials in an attractive style of writing.

Measuring the learning product.—The lack of standards and of suggestions for testing are perhaps the greatest weaknesses of the outlines examined. The outlines should contain general standards and, in so far as they may be presented with definiteness, more detailed statements of expected outcomes. These standards should be definitely in mind in suggestions made concerning tests that will objectively measure their attainment. As far as possible, objective studies should be used to give definiteness to these suggestions.

Mechanical make-up.—The latest efforts in schools in which revision has progressed over a period of years give evidence of more careful attention to the mechanical make-up of course outlines as a means of making them more attractive and usable. Outlines should be made attractive as well as forceful through use of mechanical aids. They should be made readily usable through such aids as will provide for convenience of reference. In a program of continuous revision the outlines should provide space in which teacher criticisms may be recorded.

CHAPTER II: MATHEMATICS IN JUNIOR HIGH SCHOOL GRADES

I. OBJECTIVES

Purpose of this chapter.—In the preceding chapter the general characteristics of course outlines from the selected group of schools represented in this project were considered. The present chapter is devoted to more specific consideration of the nature of objectives, content, and methods of instruction in grades 7, 8, and 9, or what we have termed the junior high school grades of the selected group of schools represented. This information has been collected from outlines, published reports, correspondence, and visitation.

Analysis of objectives.—The results of an analysis of objectives for grades 7, 8, and 9 as listed in 47 of the 57 outlines included in this project are presented in Table 7. Where objectives are given for the entire junior high school but not for the distinct grades, the same objectives are credited to each grade. All objectives are classified under seven major headings. In order that its influence may be indicated, these objectives are compared with the report of the National Committee on the Reorganization of Mathematics in Secondary Education. This committee considered all aims of secondary education as of three kinds: (1) Practical aims, which include utility of the fundamental processes of arithmetic, understanding of the language of algebra, knowledge of the fundamental laws of algebra, understanding of graphic representations, and familiarity with geometric forms; (2) disciplinary aims; (3) cultural aims.[1]

The practical aims are represented in more detail under the first four major headings of Table 7. They reveal that considerably more attention is given by the schools represented in the present study to the fundamental processes of

[1] The Reorganization of Mathematics in Secondary Education. The Mathematical Association of America (Inc.), 1923, pp. 6-10. While this report is now out of print, Houghton-Mifflin Co., Boston, Mass., published a large part of the report in 1927. This committee will be referred to hereinafter as the National Committee, and pages included in parentheses will refer to this report.

arithmetic than to algebra, graphic representations, and geometric forms. While accuracy and facility in the fundamental processes of arithmetic are supposedly well along by the time the pupil reaches grade 7, there are more objectives

TABLE 7.—*Objectives listed in course outlines in junior high school mathematics*

Objective	Grade 7 (47)	Grade 8 (47)	Grade 9 (41)
1	2	3	4
I. Accuracy and facility in the fundamental processes	40	39	35
1. Computation	24	20	17
2. Geometric skills	20	10	6
3. Understanding of fundamental laws and operations of algebra	5	10	20
4. Fundamentals	9	6	6
5. Tools of problem solving	11	5	6
6. Practical measurements	9	13	3
7. Percentage	13	1	1
8. Other	6	10	10
II. Knowledge and power to apply mathematical concepts	26	27	22
1. Concepts of mathematical law	10	10	7
2. Number sense	7	8	4
3. Symbolic notations	3	5	7
4. Mathematical terms	5	5	3
5. Other	17	13	14
III. Specific knowledge useful in life	37	34	29
1. Application of arithmetical skills	25	19	12
2. Graphs and statistics	14	8	13
3. Business office	12	13	7
4. Applications of algebra, trigonometry, and geometry	4	5	8
5. Home management	3	5	1
6. Tax and insurance	2	6	1
7. Other	6	6	
IV. Exploration and guidance	19	17	15
1. Interests and abilities	6	6	6
2. Prepare for later courses	5	5	4
3. Other	13	13	10
V. Disciplinary values	34	32	28
1. Precision in thought and statement	16	13	13
2. Self-reliance through checks	14	8	10
3. Logical reasoning	9	12	8
4. Estimate results	9	6	6
5. Quantitative relations	7	6	5
6. Discriminate true and false	4	5	3
7. Other	20	17	15
VI. Cultural values	26	30	22
1. Power of applied mathematics	9	10	9
2. Beauty of geometric design	11	7	6
3. Contribution of mathematics to civilization	7	9	8
4. Correct habits and attitudes	5	8	8
5. Interest in nature of community expense	4	5	1
6. Other	37	29	18
VII. Specific future needs of well-defined groups	8	7	8

of this type for grades 7, 8, and 9 than of any other. The exploratory values of junior high school mathematics are also emphasized to a great degree in the local outlines.

The table indicates that disciplinary values, listed under the fifth major heading, are given considerable prominence

by the schools represented. A special study of the present status of disciplinary values was included in the national report, and while a separate chapter of that report was also devoted to the functioning of mathematics in everyday life, it is significant that this is listed as an aim in only two of the local junior high school outlines examined.

Values classified as cultural are also represented in a majority of local outlines, though not so frequently as disciplinary values. Except for the development of the function concept, the objectives formulated by the National Committee seem to have exerted considerable influence on those appearing in local outlines. In one instance, however, several local committees have gone beyond the recommendation of the National Committee. The report of the committee reflects the view that differentiation due to special needs should be made after, not before, the completion of a minimum foundation in grades 7, 8, and 9, but, as indicated by Topic VII in the table, there are several cities which have included care for specific future needs of well-defined groups as one of the objectives in these grades. Under this topic were included statements indicating the intent to furnish pupils with materials related to certain future life purposes.

Emphasis on economic and social uses of mathematics are indicated in the table by the several items included under the heading "Specific knowledge useful in life." Attempts at correlation with other courses may be judged from the inclusion in all three grades of aims based on materials from algebra, geometry, and trigonometry. Attention to development of a mathematical sense as distinguished from mere mechanical manipulation is apparent in the number of aims included under "Knowledge and power to apply mathematical concepts."

Percentages calculated for the three grades show 58 per cent of all objectives classed as practical (included under the first four major headings); 20 per cent devoted to disciplinary values, 19 per cent to cultural aims, and 3 per cent to the specific future needs of well-defined groups. It is evident, therefore, that for the selected groups of schools being con-

sidered, practical or utilitarian aims of mathematics in grades 7, 8, and 9 are considered of most importance.[2]

2. COURSES REQUIRED AND ELECTIVE

The courses required and elective in the 57 systems representing grades 7, 8, and 9 are shown in Table 8. Whether subjects are required or elective is not given in many of the course outlines, but, supplemented by late programs of studies from many of the systems, the table represents practices in 44 of the 57 centers. While the National Committee recommends that mathematics be required in all three of the grades represented, the table shows this to be true for grade 9 only in slightly more than half of the schools represented. Mathematics is required for grade 7, however, in all centers and for grade 8 in all but two.

TABLE 8.—*Courses required and elective in grades 7, 8, and 9 of 44 schools or systems*

Course	Required work			Elective work *	
	Grade 7	Grade 8	Grade 9	Grade 8	Grade 9
1	2	3	4	5	6
Arithmetic	17	15			
General mathematics	14	14	5	1	8
Mathematics	12	11	2	2	5
Algebra			4	1	16
Commercial arithmetic					7
Applied mathematics					2
Junior business training					2
Clerical practice					1
Shop mathematics					1
Rapid calculation					1
Non-college preparatory mathematics					1
General mathematics or algebra			6		
General mathematics or shop mathematics	1	1			
General mathematics, algebra, or commercial arithmetic			1		
General mathematics, algebra, commercial arithmetic, or shop mathematics			1		
Arithmetic or commercial arithmetic		1	1		
Algebra or commercial arithmetic			4		
Total	44	42	24	4	44

* No elective work was reported in grade 7.

[2] Other lists of objectives of a more detailed nature which have since been published include:

Report of a Subcommittee on junior high school mathematics. North Central Association Quarterly, 2:396–420, March 1928.

Schorling, Raleigh. A tentative list of objectives in the teaching of junior high school mathematics, with investigations for determining their validity. Ann Arbor, Mich., Geo. Wahr, 1925.

Smith, David E., and Reeve, W. D. Second Yearbook, The National Council of Teachers of Mathematics, Columbia University, New York, pp. 173–227.

The subject required in grade 7 is in nearly all cases either arithmetic, general mathematics, or "mathematics." [1] With arithmetic designated in slightly more than one-third of the centers, either mathematics or general mathematics is required in well over half of all schools. Almost the same proportions exist in grade 8, but in grade 9 the distribution is more nearly equal between algebra and general mathematics, with commercial arithmetic ranking third. For the elective work in this grade, algebra is favored slightly more. The recommendation of the National Committee is for some form of correlated or general mathematics for each of these grades. Forms of applied or commercial mathematics in addition to commercial arithmetic, however, appear in the offering of 10 centers.

In another project of this survey a study of trends in 60 junior high school programs [2] indicates that for the years 1929–1931 arithmetic was required in grade 7 of 26 schools and mathematics or general mathematics in 33; in grade 8 arithmetic was required in 23 and mathematics or general mathematics in 36; in grade 9 algebra was required in 17 schools, mathematics or general mathematics in 17 schools, and commercial arithmetic in 7 schools. Mathematics was required in all schools in grades 7 and 8 and in 30 schools in grade 9. It seems, therefore, that in the more selected group of schools of this study mathematics or general mathematics appears more often.

3. CONTENT OF ARITHMETIC AND GENERAL MATHEMATICS IN GRADES 7 AND 8

Analysis of topics.—The preceding section has revealed that among the schools included in this project the number offering courses designated as mathematics or general mathematics in grades 7 and 8 is almost double the number offering courses designated as arithmetic in these grades. Just what the difference is between the content of the courses in arithmetic and in general mathematics, however, is not suggested by these titles. The traditional content of arithmetic has been established through long usage. General

[1] See Ch. V, sec. 3, of Monograph No. 19, The Program of Studies.

mathematics is defined by the committee on mathematics preparing the mathematics report for the Department of Superintendence as—

> . . . an introductory, basic, exploratory course in which the simple and significant principles of arithmetic, algebra, intuitive geometry, statistics, and numerical trigonometry are taught so as to emphasize their natural and numerous interrelations.[1]

The degree and manner in which various courses are interrelated, in both arithmetic and general mathematics, however, differ considerably. In some of the schools visited educators were found to prefer the use of the term "mathematics" to the term "general mathematics" on this account.

To indicate as far as possible the distinction in content between courses in arithmetic and mathematics or general mathematics in the schools represented, as well as to reveal the relationship between the objectives and the provisions for their realization, Table 9 is presented. Here are shown the percentages of schools offering each topic in the two courses and the average number of weeks devoted to each. The table shows the number of outlines represented in each analysis. Only a few outlines indicated the number of weeks allotted to each topic.

The topics listed in the table represent those offered in 30 per cent or more of all centers. In all instances except the arithmetic of business, ratio and proportion, and square root they represent topics suggested by the National Committee. Not all topics recommended by this committee, however, are included in 30 per cent or more of all centers. The outlines show a wide variation in the content represented, and since they vary considerably in the amount of detail included it has not always been possible to be sure of correct classification.

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[1] Department of Superintendence, Fifth Yearbook, 1927, p. 185. National Education Association, Washington, D. C.

TABLE 9.—*Topics included in seventh and eighth grade arithmetic and general mathematics in 30 per cent or more of all outlines and the average number of weeks allotted to each*

Topic	Grade 7				Grade 8			
	Arithmetic		General mathematics		Arithmetic		General mathematics	
	Percentage of outlines (13)	Average number of weeks (6)	Percentage of outlines (40)	Average number of weeks (8)	Percentage of outlines (13)	Average number of weeks (6)	Percentage of outlines (42)	Average number of weeks (8)
1	2	3	4	5	6	7	8	9
Arithmetic:								
Percentage	100	7.5	85	6.0	.....	.....	33	5.0
Fundamental operations	92	8.8	53	5.0	.....	.....	31	3.0
Business	77	5.7	73	4.3	46	3.5	38	3.8
Banking	69	5.3	60	3.5	100	4.1	43	5.4
Home	46	4.0	53	3.3	46	2.3	38	4.3
Community	.....	.....	33	.....	92	4.6	83	4.1
Investments	.....	.....	.....	.....	92	3.0	71	3.5
Fractions	46	1.5	.....	.....	.....	.....	.....	.....
Ratio and proportion	31	.....	.....	.....	54	.....	.....	.....
Square root	.....	.....	.....	.....	38	1.0	.....	.....
Graphs	62	3.0	68	2.4	38	3.0	31	3.0
Intuitive geometry:								
Direct measurement	62	3.5	78	6.4	.....	.....	.....	.....
Areas, circumferences, volumes	92	5.4	90	6.6	85	5.0	64	6.9
Numerical computations	54	.....	55	.....	.....	.....	33	.....
Indirect measurement	.....	.....	.....	.....	54	.....	48	4.0
Construction	69	2.0	60	4.6	.....	.....	.....	.....
Familiarity with forms	.....	.....	40	2.0	.....	.....	.....	.....
Algebra:								
Formula, meaning, use	.....	.....	30	3.3	46	1.0	52	2.5
Positive and negative numbers	.....	.....	.....	.....	.....	.....	43	3.3
Equation, use of	.....	.....	.....	.....	54	2.3	31	4.0

NOTE.—The numbers in parentheses indicate the number of schools, the outlines of which are represented.

We may note first the frequencies with which the different topics appear in arithmetic and in general mathematics in the seventh grade. The greatest contrasts observed are in the degree to which the fundamental operations appear as a separate topic in arithmetic and the fact that ratio and proportion and fractions are included as a topic in more than 30 per cent of the schools offering arithmetic. In the general mathematics offered in this grade, on the other hand, more intuitive geometry is taught, and algebra and community arithmetic, although not shown under arithmetic, appear in 30 per cent or more of these outlines. On the whole, how-

ever, the contrast in the topics offered in arithmetic and in general mathematics is less striking than the conception of the two would lead one to expect. It is less surprising that more intuitive geometry should be offered in general mathematics than that the large amount shown should be offered in arithmetic.

Probably not much weight can be attached to the number of weeks shown for the time allotments, since the averages are based on a few cases only and since there was considerable variation among those used. In general, however, the differences revealed are what is to be expected—more time was allotted to arithmetical topics in schools offering arithmetic and more time to intuitive geometry and algebra in schools offering general mathematics.

Order of presenting topics.—The National Committee made no recommendation as to the order in which mathematical topics should be presented. Although not presented in Table 9, averages were extracted to see just what differences would appear between arithmetic and general mathematics. These averages for the seventh grade indicate the order of presentation shown in the parallel columns which follow. The variations appearing in the outlines were wide, so that the results are not to be interpreted as dominant practices.

ARITHMETIC	GENERAL MATHEMATICS
Fundamental operations.	Percentage.
Percentage.	Graphs.
Construction work and areas.	Fundamental operations.
Circumferences and volumes in intuitive geometry.	Areas.
Banking.	Circumferences and volumes in intuitive geometry.
Fractions.	Direct measurement.
Direct measurement.	Arithmetic of business.
Arithmetic of business.	Geometrical construction.
Graphs.	Arithmetic of the home.
Arithmetic of the home.	Banking.
Numerical computations in geometry.	Familiarity with geometrical forms.
Ratio and proportion.	Arithmetic of the community.
	Geometrical computations.
	Meaning and use of the algebraic formula.

It is of interest to note that while fundamental operations are presented first in the average school offering arithmetic,

percentage and graphs are the first two topics in the average school offering general mathematics. The latter order of presentation is probably based on the theory that greater pupil interest will result from the presentation of new materials afforded by graphs than from a reconsideration of the fundamental operations of arithmetic.

In like manner the average order of topics for arithmetic and general mathematics in the eighth grade was found to be as follows:

ARITHMETIC	GENERAL MATHEMATICS
Banking.	Use of the equation.
Areas.	Areas.
Circumferences.	Circumferences.
Volumes.	Volumes.
Arithmetic of the community and of investments.	Banking.
The meaning and use of the algebraic formula.	The meaning and use of the algebraic formula.
Ratio and proportion.	Arithmetic of the community.
Indirect measurement.	Indirect measurement.
Use of the equation.	Fundamental operations.
Arithmetic of the home.	Arithmetic of business.
Graphs.	Graphs.
Arithmetic of business.	Percentage.
	Arithmetic of the home.

In a comparison of the nature of topics in grade 8 with those in grade 7, data for the courses in arithmetic and in general mathematics reveal, on the whole, similar results, except that the emphasis in general mathematics for the higher grade is more on algebra than on intuitive geometry. In the schools offering arithmetic, ratio and proportion and square root have continued as major topics, although these topics are recommended only for incidental treatment in the national report. The same contrast appears in the order of presenting topics—the use of the algebraic equation averages first for general mathematics courses, while the arithmetic of business is first for arithmetic courses. Little distinction is found in time allotments, except that on the basis of the few schools represented the allotment is shown to be greater for practically all topics in schools offering general mathematics.

Social uses emphasized.—The preceding analysis indicates that for both arithmetic and general mathematics considerable emphasis is being placed upon practical economic and social uses of arithmetic. Such topics as the arithmetic of business, banking, the home, the community, investments, graphs, direct measurement, and the meaning and use of the algebraic formula appear with slightly more frequency in general mathematics, but with considerable frequency in arithmetic as well. From a statement of topics, however, it was not possible to judge with any certainty as to the amount of emphasis placed on the development of disciplinary and cultural aims.

Course outline at Rochester.—Among the schools visited, a most carefully constructed course of study for intuitive geometry was noted in that outlined for the seventh grade at Rochester, N. Y. The topics in arithmetic and geometry are interspersed throughout the grade, as is indicated in the following adaptation of the "tentative time schedule":

7–B

1. Ten lessons to daily reviews of integers, fractions, and decimals in connection with the introduction of the following topics in intuitive geometry: (1) Origin of mathematics; (2) why we study geometry; (3) geometry of form; (4) geometry of size.
2. Ten lessons to percentage: Meaning, importance, common equivalents.
3. Five lessons to (1) the geometry of position; (2) geometric ideas and their importance.
4. Ten lessons to percentage.
5. Eight lessons to geometry: Points and lines, direct measurement of segments.
6. Five lessons to simple applications of percentage.
7. Ten lessons to arithmetical statistics and geometrical graphs.
8. Eight lessons to business applications of percentage.
9. Seven lessons to the circle.
10. Five lessons to the application of percentage to life situations.

7–A

1. Ten lessons to the angle.
2. Eight lessons to percentage: Use of decimal and fractional per cents.
3. Five lessons to circles, angles, and parallel lines.
4. Five lessons to arithmetical statistics and geometrical graphs.

5. Six lessons to the arithmetic of business.
6. Ten lessons to the triangle.
7. Fifteen lessons to thrift and banking.
8. Five lessons to direct measurement of perimeters, areas, and angles.
9. Ten lessons to everyday applications of percentage.
10. Ten lessons to symmetry, fundamental constructions, and applications.

The course of study is introduced by giving separately for arithmetic and geometry an introductory statement concerning the educational significance of the subject, central objectives, and a general outline of all topics included in the course. After a tentative time schedule each of the topics is treated separately with respect to detailed suggestions for motivation, development, and application, in which are included classroom procedures, summary of outcomes, and suggested tests. In the appendix are included (1) an index of important terms arising in intuitive geometry; (2) a summary of important geometric facts and relations.[5]

The eighth-grade course in the same city follows in general the same order of presentation. In 8–B geometry is continued, but in grade 8–A major emphasis is on algebra with correlation and review of arithmetic and intuitive geometry.

8–B

1. Ten lessons to congruence and similarity in geometry. Reviews in arithmetic.
2. Ten lessons to indirect measurement in geometry. Reviews in arithmetic.
3. Eight lessons to indirect measurement correlated with ratio and proportion.
4. Eight lessons to the hypotenuse rule in geometry correlated with square root.
5. Fifteen lessons to banking. Geometry reviews.
6. Ten lessons to measurements (perimeters, areas, volumes). Arithmetic reviews.
7. Twelve lessons to investments. Geometry reviews.
8. Five lessons to statistics and graphs correlated.
9. Five lessons to taxes. Geometry reviews.

[5] See also Betz, William. The Teaching of Direct Measurement in the Junior High School. Third Yearbook, The National Council of Teachers of Mathematics. Columbia University, New York, pp. 149–194.

8–A

1. Fifteen lessons to first steps in algebra. Arithmetic: Fundamental operations (integers, fractions, decimals, used in addition of similar terms and evaluation). Geometry: Measurement of perimeters (used in formulas).

2. Fifteen lessons to the use of formulas. Arithmetic: Fundamentals; business situations. Geometry: Measurement of perimeters, areas, volumes.

3. Ten lessons to the making of formulas. Arithmetic: Fundamentals (used in evaluation). Geometry: Polygons.

4. Ten lessons to the equation. Arithmetic: Fundamentals (used in checking).

5. Ten lessons to problem solving. Arithmetic: Fundamentals; percentage applications; business problems. Geometry: Perimeters, areas, volumes.

6. Ten lessons to graphs of formulas; graphic solution of problems. Arithmetic: Statistics. Geometry: Bar, line and circle graphs.

7. Fifteen lessons to signed numbers. Arithmetic: Fundamentals; simple problems. Geometry: Graphic representation of signed numbers.

Textbook analyses.—Since, as shown in Table 5, a majority of the centers being considered connect their outlines to some extent with the textbooks in use, illustrations of topics treated as revealed through analyses of textbooks will be of interest in this connection. In a study published in 1929, among the comparisons made, McCormick compares the results of an analysis of 11 seventh and eighth grade arithmetics published since 1925 with the results of a similar analysis of 10 junior high school textbooks published since 1923. The degree to which different topics are treated in the two analyses he summarizes as follows:

> The junior high school textbooks for the grades now being considered contained about the same amount of statistical graphs as the books written for the 8-year elementary schools. In every other branch there was a very noticeable difference. Arithmetic dropped from an average of about 80 per cent in the arithmetics for elementary schools to about 59 per cent in the junior high school textbooks. Algebra increased from 5.5 per cent to 15.3 per cent and intuitive geometry from 16 per cent to 21.3 per cent. There was a trace of algebraic graphs in all the junior high school textbooks investigated except one and also a little numerical trigonometry in 3 of the 10 books.[6]

[6] McCormick, Clarence. The Teaching of General Mathematics in the Secondary Schools of the United States. Teachers College, Columbia University, Contributions to Education, No. 386, Ch. V. New York, 1929, p. 61.

In a more detailed but limited analysis of textual materials, Mrs. David R. Davis reports evidence obtained through an examination of 10 series of three textbooks each which were published within the last eight years.[7] She shows that 8 of the 10 authors offer materials from arithmetic, geometry, and algebra in the seventh-grade course, and each of the 10 presents materials from these same subjects in the eighth grade. For grades 7, 8, and 9 the range in percentage of space devoted to the separate subjects is as follows: Arithmetic, 26.10 to 58.89; algebra, 33.58 to 57.61; geometry, 11.97 to 40.99; trigonometry, 0 to 12.66; graphs, 9.85 to 8.54. Among Mrs. Davis's conclusions the following are noteworthy: (1) Much use is made of material for motivating all phases of the work; (2) a tendency is noted to present work in a psychological rather than a logical manner; (3) mechanical problems are given much less space than verbal problems; (4) a tendency is present to develop practical rather than impractical mathematics; (5) much effort is devoted to training the child for quantitative interpretation.

These two analyses of textbooks confirm on a more general scale the results presented in Table 9. Mathematics in grades 7 and 8 is more inclusive than it was 10 years ago, and the exploratory functions of the junior high school are being carried into this field.

4. ADAPTATION TO PUPIL NEEDS AND INTERESTS IN GRADES 7 AND 8

The chief problem.—The problem in seventh and eighth grade mathematics over which teachers in the schools visited seemed most concerned was the development of interest in the practical application of mathematics without the sacrifice of such drill as is necessary for accuracy and facility in the fundamental processes. The belief in the necessity of giving a considerable number of seventh-grade pupils more than a brief review of the fundamental processes was current, but the attainment of practical results that could not be secured from routinized drill alone was also desired.

[7] Davis, Mrs. David R. A Comparative Study of Textbooks in Junior High School Mathematics. The High School, 8 : 103-114, February, 1931.

Motivation.—Adequate motivation obviously plays a large part in the solution of this problem. The history of mathematics, in which the necessity of counting and measuring are shown to have brought about the development of arithmetic and geometry, is very carefully worked into the outline for these grades at Rochester. Many ways of making these ideas more real through actual contact of the child with his environment are put into effect. Such activities as drawing, simple construction, direct measurements, observing natural forms and positions, visits to local civic and business enterprises, securing concrete illustrations from the home, and the like are made a part of the regular classroom procedure. These activities are planned not only to create greater interest but to bring about a natural correlation of different branches of mathematics as well.

Problem solving at Cleveland.—After installing a new junior high school course of study in 1925, the department of research of the Cleveland schools began periodic checking to measure results. By 1927 they were fairly well satisfied with the degree to which all objectives were being realized except that of problem solving. Experiments reported in mimeographed bulletins showed that dynamic or real life problems caused a sharp rise in achievement. After the pupils are taught to recognize and write problems illustrating the practical uses of arithmetic, they are required to collect such problems from real situations, to exchange, and to solve them. This procedure is believed to be responsible for later realization of adequate norms on this project, as was shown on tests given to all pupils enrolled in grades 7 and 8 and in the lowest ability group in grade 9. A number of these problems are published in booklet form. Activities for the development of the social and economic uses of arithmetic are especially well planned at the Fairmount Junior High School in Cleveland.

Motivation at John Burroughs School.—Many plans which contribute to motivation of seventh and eighth grade mathematics classes are to be found in the John Burroughs School at Clayton, Mo. While very much committed to developing the exploratory functions of the junior high school, the staff is also convinced that pupils in these grades need guidance

in integrating the many bits of new subject matter to which they are introduced. Two years ago a unit course was introduced in which every seventh-grade pupil is required to spend one and one-half hours per day. Several teachers contribute to this course, which is devoted to a somewhat dramatic presentation of how man made nature to serve him in early times. In mathematics the class traces the growing needs in number and measurement as the mental life of man expanded. One of the activities of the class was the staging of a pageant entitled "A History of Numbers." The results of this course have been so encouraging that the staff is now engaged in organizing a similar course which will lead through the progress of civilization up to the machine age, and will be required of all eighth-grade pupils.

At the time of the writer's visit a class of seventh-grade pupils, with chairs grouped closely in a semicircle before the teacher, was much concerned over the accurate construction of arcs and circles. One reason why geometric construction was considered at this time was that this group then had a project in constructing a church window in their art class. They had need of accuracy in measuring, since they were actually to make the measurements. Different pupils went to the blackboard and with chalk and string and suggestions from the class attempted the constructions needed. All seemed deeply interested and some were making trials at their desks. At the end of the class period they all agreed that they would hand in more accurate constructions at the beginning of the next day's recitation.

Drill.—In several of the centers visited efforts are made to conduct drill in such a way that the course of study does not become too mechanical or tiresome for the pupil. In three of these centers the schedule provides that 10 minutes of each period be devoted to drill. Among the junior high schools visited special drill books or practice materials had been prepared and published by members of the staff at Cleveland, Rochester, the University of Chicago Laboratory School, and the John Burroughs School.

Development of a "mathematical sense" at Boston.—In Boston, drill is connected in many ways with an aim which both junior and senior high school teachers seem constantly

to keep in mind, namely, the development of a "mathematical sense."[8] Wherever possible, they attempt to have the pupil take a common-sense view of the application of mathematics to practical situations. The application of this aim to the development of accuracy was illustrated in the classrooms visited in three major ways: (1) Reverse multiplication; (2) checking all results; (3) estimating the answer.

The common-sense view which reverse multiplication furthers is well illustrated in problems of measurement involving decimals. For example, to find the area of a lot 74.6 feet long and 45.3 feet wide involves the following:

(1) Estimate: $75 \times 45 = 3375$.

(2) Reverse multiplication:

74. 6 ft.
45. 3 ft.

2984.
373. 0
22. 4 (0.38=0.4 approx.)

3379. 4 sq. ft.

(3) Check:

45. 3 ft.
74. 6 ft.

3171.
181. 2
27. 2 (0.18=0.2 approx.)

3379. 4 sq. ft.

Reverse multiplication.—In the computation by reverse multiplication, since there is one decimal place in each figure, the complete answer will obviously have two decimals. It is impossible, however, to measure with accuracy to hundredths of a square foot, and for this reason the product is made to show only tenths of a square foot. The pupils are further required to make an estimate of their results in all cases and to check all answers, as a means both of contributing to their sense about number and to their accuracy.

5. CONTENT OF ALGEBRA AND GENERAL MATHEMATICS IN GRADE 9

Analysis of topics.—A topical analysis, similar to that shown for grades 7 and 8 in Table 9, was also made of 28 courses in algebra and 24 courses in general mathematics, as offered in grade 9. These data, showing the percentage of schools offering the various topics, the order in which the topics are presented, and the average number of weeks

[8] The need of this is emphasized in an article by Charles H. Judd. Informational Mathematics versus Computational Mathematics. Mathematics Teacher, 22: 187-196, April, 1929.

allotted to each, are shown in Table 10. Data for time allotments, however, were secured from only 14 algebra courses and 3 general mathematics courses. Again, difficulty was encountered, especially in general mathematics courses organized on the unit plan, in classifying, from the name of the unit only, the topic under which it belonged. For this reason the percentages shown for topics, particularly in schools offering general mathematics courses, is probably lower than would be the case if some outlines had been more detailed.

TABLE 10.—*Topics included in ninth-grade algebra and general mathematics in 30 per cent or more of all outlines, the order in which presented to the class, and the number of weeks allotted to each*

Topic	Algebra			General mathematics		
	Per centage of outlines showing (28)	Average of the order in which presented (28)	Average number of weeks of time allotted (14)	Per centage of outlines showing (24)	Average of the order in which presented (24)	Average number of weeks of time allotted (14)
1	2	3	4	5	6	7
Fundamental operations of algebra	89	1	4.9	54	5	5.0
Factoring	86	4	4.4	67	8	4.5
Fractions	86	6	5.5	67	7	4.5
Exponents and radicals	86	9	3.4	42	13	3.0
Equation, its use	79	2	3.8	54	3	3.3
Simple quadratics	71	10	2.7	50	19	2.5
Graphs and graphical representations	71	7	1.9	50	4	1.0
Positive and negative numbers	68	3	1.7	71	2	.....
Linear equations	64	8	2.8	42	9	3.0
Formula, meaning and use	57	5	1.0	79	1	2.3
Application of ratio and proportion	39	11	2.0	58	10	2.0
Equations in two unknowns	30	12	3.0	38	21	3.0
Trigonometry	.....	.....	.....	54	38	5.0

NOTE.—The numbers in parentheses indicate the number of outlines represented.

Although differences are not as great as for grades 7 and 8, the table reveals considerable variation in the percentages of schools in which topics are offered in general mathematics and in the traditional algebra in grade 9. In the First Yearbook of the National Council of the Teachers of Mathematics,[9] the meaning and use of the formula, graphs and

[9] Smith, David Eugene. A General Survey of Progress of Mathematics in Our High Schools in the Last 25 Years, pp. 1–31.

graphical representations, directed numbers, linear equations, and numerical trigonometry are suggested as the five topics of central importance. Of these five, the first and third ranked within the five highest percentages of schools offering in the general mathematics group, but none of them attained such rank in the schools offering algebra. While the percentages vary considerably for the two courses, such traditional topics as fundamental operations, factoring, and fractions continue to occupy high rank in each.

Considerably more variation in the order in which topics are introduced is rather to be expected, since there have been no central recommendations along this line. In view of the fact that the general mathematics courses have as one of their purposes the organization of materials along more psychological lines, it is significant that the meaning and use of the formula attains first rank in this list as contrasted with fundamental operations, which ranks first in algebra.

Probably not a great deal of significance can be attached to the time allotments shown, since so few centers are represented. The figures presented in the table indicate a more liberal allotment to the traditional topics offered, more largely in algebra. The most significant differences revealed are in the greater amount of time allotted to fractions in algebra and to the meaning and use of the formula in general mathematics.

Analysis of textbooks.—Comparisons of the content of courses in algebra and in general mathematics are also made through studies in which textbooks of such titles have been analyzed. In the study made by McCormick (p. 71), twelve textbooks in algebra published since 1923 are compared with eight textbooks in general mathematics for the ninth grade. The mean percentages which he computed for the different branches are presented in Table 11.

The summary indicates that considerably more attention is devoted to algebra and a little more to trigonometry in textbooks in algebra, but that more attention is given to arithmetic, statistics, and geometry in textbooks in general mathematics.

TABLE 11.—*Mean percentages by which different branches are included in certain textbooks in algebra and general mathematics, grade 9*

Branch	Mean percentage for—	
	Algebra	General mathematics
Arithmetic	5.0	5.3
Statistics, computational	.3	1.1
Statistics, graph	3.9	3.6
Algebra	76.0	58.2
Algebra, graph	4.6	5.3
Geometry, intuitive	6.2	22.5
Geometry, demonstrative	0	1.8
Trigonometry	3.9	2.1
Total	99.9	99.9

Another study in which changes in the content of elementary algebra from 1818 to 1928 are considered is that of Chateauneuf,[10] in which 257 textbooks were analyzed. Her data are presented by 10-year periods and show that the most significant changes between the periods 1910–1920 and 1920–1928 are an increase in the percentage of space devoted to general equations and formulas and a decrease in the space devoted to factoring, factors, and multiples. In comparing changes of content materials from 1890 to 1928 with the recommendations of the National Committee, she shows that they are favorable along the following lines:

1. An increase in both number and percentage of verbal exercises.
2. A decrease in the percentage of tool exercises (although there was an increase in the mean number).
3. A decrease in the emphasis on complicated algebraic technique.
4. Limitation of drill to certain processes deemed necessary by the committee.

It seems that both with regard to the selected group of schools and with regard to textbooks in elementary algebra, changes from traditional organization of seventh and eighth grade mathematics have not been so great as in content.

[10] Chateauneuf, Amy Olive. Changes in the Content of Elementary Algebra Since the Beginning of the High-School Movement as Revealed by the Textbooks of the Period. University of Pennsylvania, 1929, p. 140 ff.

6. ADAPTATION TO PUPIL NEEDS AND INTERESTS IN GRADE 9

Required work.—The recommendation of the National Committee regarding the required work of grade 9 is contained in the following quotation (p. 14):

> The National Committee believes that the material described in the next chapter should be required of all pupils and that under favorable conditions this minimum of work can be completed by the end of the ninth school year. In the junior high school, comprising grades 7, 8, and 9, the course for these three years should be planned as a unit with the purpose of giving each pupil the most valuable mathematical training he is capable of receiving in those years, with little reference to courses he may or may not take in succeeding years. In particular, college entrance requirements should, during these three years, receive no specific consideration.

The materials recommended are taken from arithmetic, algebra, intuitive and demonstrative geometry, and trigonometry. Arithmetic is recommended which applies "particularly in such lines as relate to commercial, industrial, and social needs."

As shown in Table 8, of the 44 schools represented, only 7 require general mathematics in grade 9; 8 require algebra or either algebra or commercial arithmetic; 6 require algebra or general mathematics; and 2 allow choice among general mathematics, algebra, and some form of applied mathematics. Altogether, some form of mathematics is required in grade 9 in 55 per cent of the 44 schools. In the schools requiring mathematics, such requirement must be met with general mathematics in 63 per cent of the schools; algebra in 64 per cent; and with some form of applied mathematics in 29 per cent of the schools.

While algebra is still the course offered most often in the ninth grade, percentages compiled for the United States as a whole by the Office of Education reveal that the proportion of pupils pursuing this subject has been decreasing since 1905. Between 1922 and 1928 there was a decrease in algebra of from 40.15 per cent to 35.22 per cent of all pupils enrolled in public high schools. [11]

The problem in the ninth grade.—Despite modifications from traditional content which the analyses of topics have

[11] Biennial Survey of Education. U. S. Office of Education, Bulletin, 1930, No. 16, p. 1057.

shown to be offered in grades 7, 8, and 9, the evidence of the preceding paragraph, together with that secured from the schools visited, seems to point to the fact that the adaptation of ninth-grade mathematics to the interests and needs of the student body is the chief problem faced in the three grades being considered.[12] Added to the recommendation of the National Committee that mathematics be required of all ninth-grade pupils is that of the mathematics committee preparing the report for the yearbook of the Department of Superintendence.[13] This committee, after presenting topical outlines of ninth-grade mathematics from two cities, one of which adheres rather closely to algebra, while the other admits more of the economic uses of arithmetic, recommends that the former be considered ideal, but that the latter be used preliminary to reaching the stage when more algebra can be taught.

Difficulties in cities visited.—The problem faced by curriculum committees in mathematics in several of the cities visited is the selection of materials for grade 9 which will meet the needs and interests of pupils going to college as well as those of pupils not needing the same amount of algebra. This difficulty appears most pressing in schools which require ninth-grade mathematics of all pupils. In four of the public schools visited this requirement was made. In a fifth city, while not required, limitation in the choice of electives forced almost all pupils to take ninth-grade mathematics. Three of these five cities considered the adaptation of subject matter to the needs of the non-college-going pupil as distinguished from the college-going pupil as their most urgent problem in the field. While this problem was not mentioned in the other two cities, one of them is suburban to a large city with most of its pupils preparing for college, and in the other sufficient time had not elapsed for securing the reactions of teachers to a new course of study.

The course required of all pupils in each of these cities is algebra, rather than general mathematics. In one case the change had just been made to a textbook containing more

[12] See also Betz, William. Whither Algebra?—A Challenge and a Plea. Mathematics Teacher, 23: 104-125, February, 1930.

[13] Fifth Yearbook, Department of Superintendence, National Education Association, Washington, D. C., Ch. XI.

algebraic content in order to give a more serious challenge to the college-entrance group. In the other two schools the course of study as taught up to 1931 caused too many failures among the slower pupils. In each instance the solution was expected to be the offering of a choice between general mathematics and algebra instead of offering one course for all pupils.

Some teachers of schools visited felt that of the pupils who came to them in the ninth grade not all are capable of understanding algebra. A class of seventh-grade pupils was visited at the James V. Thompson Intermediate School in Boston in which the methods pursued in introducing the class to equations seemed calculated to guard against just such criticism. Pupils were made to feel that a new method—solving by equations—was being introduced because it might prove to be an easier method of solving problems in discount on which they were then working. The necessity, in making equations of having one side equal to the other was explained in terms of a weighting scale, the sides of which must be made to balance. The principles of mathematics involved were brought out through illustrations familiar to the pupils whenever occasion arose in order to avoid the mechanical side of algebra as much as possible.

The ability to generalize is the problem which pupils face to a greater degree in algebra than in their previous mathematical experience. Training of this nature is made a very definite objective in grades 7 and 8, as well as in grade 9, in some of the schools visited. At the John Burroughs School the division of problems into types, with a presentation of the facts and relationships common to such types, has proved effective in teaching pupils to solve verbal problems.[14] Reference has been made to the plan through which the norms of Cleveland pupils in problem solving were achieved. An attempt has been made by Powell to secure from teachers of mathematics a group of problems which will be more real to the pupil and challenge their interest to a greater degree.[15]

[14] Haertter, L. D. An Effective Method of Teaching Pupils How to Solve Verbal Problems. Mathematics Teacher, 24: 166-175, March, 1931.

[15] Powell, Jesse Jerome. A Study of Problem Material in High-School Algebra. Teachers College Contributions to Education, No. 405, Columbia University, New York City, 1929.

Measuring results.—Few of the outlines examined contained detailed achievement standards for each semester or grade. The Bureau of Educational Reference and Research of the University of Michigan publishes An Outline of Essentials in Junior High School Mathematics which is based on the course of study of the University High School at Ann Arbor. For each semester of each grade the achievements expected of the pupils are outlined under the following headings: Vocabulary, Principles, Skills, Formulas, Concepts, and Attitudes.[16]

Drushel calls attention to the need "that we receive pupils entering junior high school where they actually are in their mathematical experience, not where the courses of study says they ought to be, nor where we who are to be their teachers think they ought to be. . . ." [17] Diagnostic testing occupies a very important place in the mathematics program at the University of Chicago High School. Classes are conducted on the laboratory plan and each teacher feels keenly the responsibility of knowing the weaknesses and strength of each pupil and of directing remedial work to such needs. In this school, enrolling about 500 pupils, approximately 75 per cent are enrolled in mathematics, although only one year of the upper four years' work is required. The function concept plays a very important part in the methods of instruction of grades 9 to 12. Breslich, head of the department there, calls attention to the manner in which this recommendation of the National Committee is slighted in modern textbooks.[18] He has devised an 11-page test to measure functional thinking of pupils in grades 9 to 12 of the University High School. The test is divided into eight parts, as follows: I, Recognizing Relationships; II, How a Change of one Variable in a Formula Affects the Others; III, Interpreting Graphs; IV, Changes in Equations; V, Reorganizing Linear and Quadratic Equations; VI, Depend-

[16] See also Schorling, Raleigh. The Need for Being Definite with Respect to Achievement Standards. Mathematics Teacher, 24: 311-320, May, 1931.

[17] Drushel, J. Andrew. Important Contributions of the New Mathematics with Special Reference to the Junior High Grades. Junior-Senior High School Clearing House, 5: 332-335, February, 1931.

[18] Breslich, Ernest R. Functional Thinking. Third Yearbook, National Council of the Teachers of Mathematics, Columbia University, New York, pp. 42-56.

ence in Formula; VII, Variation; VIII, Expressing Relationships in Mathematical Symbols. There is no doubt that constant focusing of the attention of the pupil on the dependability of numbers on one another helps to develop reasoning in the thinking of the mathematics pupils in this school.

CHAPTER III : MATHEMATICS IN SENIOR HIGH SCHOOL GRADES

1. OBJECTIVES

Decrease in percentage of enrollments.—Changes in mathematics in grades 10, 11, and 12 have not been so widespread as those indicated for the grades usually included in the junior high school organization. That pupils in late years are not choosing mathematics so readily is suggested from a comparison of the enrollments in mathematics in these upper grades over a period of years. While the *number* of pupils enrolled has, of course, increased, figures compiled by the United States Office of Education for 1922 and 1928 reveal decreases in late years in the *percentage* of pupils enrolled in the courses usually offered in grades 9 to 12, as follows:[1] Advanced arithmetic, 10.53 to 2.43; algebra, 40.15 to 35.22; geometry, 22.68 to 19.80. Slight increases are shown in trigonometry and in commercial arithmetic, but even the latter increase does not cover the decrease shown for advanced arithmetic.

While the objectives formulated by the National Committee as considered in the preceding chapter are applicable to the upper years of secondary education as well, this committee directed attention to the criteria which should govern the selection of materials for these grades: (1) Some attention to pupils' vocational or other later educational needs; (2) mathematical ideas and processes that have most important applications in the modern world included as far as possible; (3) increased attention to logical organization.[2]

Only 17 outlines of the 46 examined listed objectives that might be considered as applicable to the upper three or four years of secondary education. These objectives, analyzed on the same basis as those for grades 7, 8, and 9, are presented in Table 12. As might be expected, aims pointing to practical

[1] Biennial Survey of Education, 1926-1928. U. S. Office of Education, Bulletin, 1930, No. 16, pp. 1057-1058.

[2] The Reorganization of Mathematics in Secondary Education. The Mathematical Association of America (Inc.), pp. 32-33. This committee will be referred to hereinafter as the National Committee, and pages indicated in parentheses will refer to this report.

or utilitarian values of mathematics are not listed so often in the upper as in the lower grades, while those pointing to the disciplinary and cultural values appear more often. Neither is it surprising that the teaching of geometry should cause emphasis on the cultivation of such disciplinary values as precision, accuracy, logical structure, and reasoning. It is, however, again to be noted that less than half of the outlines indicate emphasis on functional thinking or thinking in terms of the relationships between quantities.

TABLE 12.—*Objectives for mathematics in grades 10, 11, and 12 in 17-course outlines*

Objective	Number of schools
I. Accuracy and facility in fundamental processes	12
Computation	5
Geometry	4
Problem solving	3
Algebra	3
II. Knowledge and power to apply mathematical concepts	8
Mathematical law	4
Use symbolic notations	3
III. Specific knowledge useful in life	5
Understand and interpret graphs	5
IV. Exploration and guidance	3
V. Disciplinary values	15
Mental habits: Precision, accuracy, etc	10
Thinking in terms of relationships (function)	7
Logical reasoning	6
Logical structure	5
Discriminate true and false	4
Quantitative relations	4
VI. Cultural values	17
Mathematics to civilization	6
Beauty of geometric design	6
Power of applied mathematics	4
Habits and attitudes	4
Originality	4
VII. Specific future needs of well-defined groups	7

8. COURSES REQUIRED AND ELECTIVE

The courses recommended by the National Committee for grades 10, 11, and 12 are plane demonstrative geometry, algebra, solid geometry, trigonometry, elementary statistics, elementary calculus, history, and biography, and additional

electives, such as shop mathematics, surveying, navigation, and the like, in schools having special needs (pp. 34–39). None is recommended to be required of all pupils. The required and elective courses in 43 of the schools whose outlines are represented are presented in Table 13. The traditional college-preparatory subjects—plane and solid geometry, algebra, and trigonometry—are offered in practically all schools, but statistics, calculus, history, and biography are not offered at all under these names. As will be seen later, however, materials from these fields are sometimes presented in connection with other courses.

TABLE 13.—*Courses required and elective in grades 10, 11, and 12 of 43 schools*

Course	Required work [1]		Elective work		
	Grade 10	Grade 11	Grade 10	Grade 11	Grade 12
1	2	3	4	5	6
Plane geometry	3	2	32	11	
Solid geometry				17	24
Algebra	1		5		
Plane geometry or commercial arithmetic	1				
Algebra, commercial arithmetic, or shop mathematics	1				
Intermediate algebra				8	
Advanced algebra			2	24	17
College algebra					6
Trigonometry				1	38
Commercial arithmetic			4		
Shop mathematics			4	4	2
Related mathematics			3	1	3
Applied mathematics			1		
Vocational mathematics			1	1	1
Mathematics			3	2	3
Engineering mathematics					1
Social arithmetic					1
Mathematical analysis					2
Mathematics review					1
Advanced arithmetic				1	2
Plane surveying				1	
Total	6	2	55	71	101

[1] No required work was reported in grade 12.

Mathematics is required in the tenth grade in six schools, in the eleventh grade in two schools, and in the twelfth grade in no schools. It is of some significance that plane geometry is required in only 5 of the 43 schools represented. This is evidence that schools of this study are a selected group. In another section of the survey involving a study

of senior high school programs, 21 of the 52 schools which were chosen at random required plane geometry of all pupils.[3]

Plane geometry is usually offered in the tenth grade, while solid geometry and advanced algebra are rather equally divided between grades 11 and 12. In schools in the New England and Middle Atlantic States intermediate algebra is often offered in grade 11 and advanced algebra in grade 12. In the Middle West, however, algebra work is offered in the same years, but designated as advanced algebra in each year. Trigonometry is offered in grade 12 in all cases but one.

The table reveals little variation from the way in which mathematics has been offered in the upper grades for many years, except in the lesser number of schools requiring courses. Some variations which the table does not specifically reveal, however, will be brought out in other connections.

3. MATHEMATICS IN GRADE 10

It is indicated in Table 13 that, in the centers whose outlines are considered, the tenth-grade offering consists of plane geometry in 32 systems, algebra in 7, and mathematics related to some particular field, such as commercial arithmetic and shop mathematics, in 13. In other words, the mathematics offering appears largely college preparatory. More specific examination, however, reveals certain changes from the traditional college-preparatory offering. A few schools offer two years of algebra before the pupil is introduced to demonstrative geometry, and at the Public Latin School in Boston geometry and algebra are offered as parallel courses in grade 10, the five class periods of each week being divided rather equally between the two courses. It is claimed that the algebra-geometry-algebra sequence, attended with a minimum use of algebra in geometry, causes considerable loss, and that efforts are needed to remedy adjustment difficulties between the two courses.[4]

There are other adjustments besides that with algebra which are called for in the traditional offering in plane

[3] See Ch. XIV, sec. 4, of Monograph No. 19, The Program of Studies.
[4] Hart, Howard F. Mathematics Teacher, 24: 177-179, March, 1931.

geometry. Certain leaders in mathematics, noting the small number of pupils who take solid geometry, are advocating methods of reorganization through which some training in 3-dimensional space may be secured by a larger number of high-school graduates. The National Committee suggested that with the introduction of intuitive geometry in the earlier grades such saving of time might result in the covering of a minimum course in plane geometry in one-half a year (p. 35). A committee was appointed by the Mathematics Association in 1929 to consider with the college entrance board the feasibility of modifying college entrance examinations to include the essentials of plane and solid geometry in one year.[5]

In two of the course outlines represented in this project are to be found reorganizations of the content of the plane-geometry course with this end in view. The 1930 outline prepared for Oakland, Calif., provides, in addition to plane geometry, for the inclusion of materials from solid geometry, and some suggestions for correlating trigonometry and algebra with the tenth-grade offering. No formal proof in solid geometry, however, is contemplated.[6] The Senior High School Course of Study of 1927 for Denver, Colo., however, includes the proof of theorems in plane and solid geometry in the outline for tenth-grade pupils. Such inclusion involves the elimination of formal proof of a number of propositions in each of the two fields. Both at Denver and at Oakland, however, another half year's work in plane and solid geometry may be taken later in high school.

Several other suggested outlines for 1-year courses in plane and solid geometry have appeared in print. They vary between "tandem" courses, in which solid geometry is added after plane geometry has been finished, to suggestions

[5] Report of the Committee on Geometry. Mathematics Teacher, 24:298-302, May, 1931.

[6] See also Allen, Gertrude E. Fifth Yearbook, The National Council of Teachers of Mathematics, Columbia University, New York. An experiment in redistribution of material for high-school geometry, pp. 67-85.

for a complete fusion.[7] The reports received from teachers who have experimented with reorganization have not been encouraging. While portions of solid geometry may no doubt be given intuitively with the regular plane-geometry course, no schools have reported what they considered a successful completion of the two courses in one year.

The foregoing illustrations, however, evidence the attack being made in some quarters on traditional content in plane geometry. Changes of a less fundamental nature will be indicated in the present chapter, the remainder of which is devoted largely to instruction in plane geometry.

4. OBJECTIVES OF PLANE GEOMETRY

Of the courses in which objectives for plane geometry are listed, the following appear in five or more:

1. Development of logical reasoning (9 outlines).
2. Appreciation of utility and beauty of geometric forms (7).
3. Familiarity with properties, mensuration, and relationships of common geometric forms (6).
4. Understanding and appreciation of deductive proofs (6).
5. Understanding of spatial concepts and relations (6).
6. Precision and accuracy (5).
7. Appreciation of the part geometry has played in the development of civilization (5).

Besides emphasizing the intuitive phases of geometry, the objectives indicate that teaching in this field is largely to be devoted to the development of logical thinking.

The National Committee, in addition to its recommendations concerning emphasis on logical organization, the study of intuitive geometry in the early grades, and atten-

[7] For suggestive courses, refer to the following:

Austin, C. M. Report of Second Committee on Geometry. Mathematics Teacher, 24: 370–394, October, 1931.

Beatley, Ralph. Demonstrative Geometry. Mathematics Teacher, 24:213–222, April, 1931.

Evans, George W. Report of Second Committee on Geometry. Mathematics Teacher, 23:87–94, February, 1930.

Orleans, Joseph B. The Fusion of Plane and Solid Geometry. Mathematics Teacher, 24:151–159, March, 1931.

Reeve, W. D., and others. Tenth-year Mathematics Outline. Mathematics Teacher, 23:343–357, October, 1930.

Stone, Charles A. A Combined Course in Plane and Solid Geometry. Mathematics Teacher, 24:160–165, March, 1931.

tion to the development of the function concept, has made the following suggestions on the teaching of geometry (p. 49):

1. More frequent use of the idea of motion in demonstration.
2. Omission of formal treatment of theory of limits and of incommensurable cases.
3. Through effective organization of those taught, a reduction in the number of theorems given.

The geometry course outlines of 44 schools were examined to ascertain whether these items and a few others were specifically brought out. The results are as follows:

Emphasis on logical organization	32
Intuitive geometry in earlier grades	7
Development of function concept	3
More frequent use of idea of motion	1
Omission of theory of limits and incommensurables	14
Reduction in number of theorems	22
Emphasis on treatment of originals	17
Suggestions as to use of algebraic method	4
Closer coordination with trigonometry, solid geometry, or calculus	14
Independence of textbooks	11
Suggestions as to distinct courses for separate groups	3

In the majority of the outlines emphasis on the logical organization and the development of logical reasoning are stressed, and in exactly one-half there is indication that the regular number of theorems may be reduced. Otherwise, the indicated items are not specifically stressed in one-half the outlines. It is realized, of course, that these items may be stressed in the classroom even though no particular attention is given them in the course of study outline; yet their importance in the minds of the committee is indicated by whether or not the outline calls specific attention to them.

5. ANALYSIS OF TEXTBOOKS IN PLANE GEOMETRY

The criticisms against traditional geometry and the extent to which they have been met through textbooks are carefully analyzed in a recent study of trends by Miss Freeman.[8] After summarizing recurrent criticisms against prevailing practices in the teaching of geometry the author

[8] Freeman, Ellen M. Textbook Trends in Plane Geometry. School Review, 40:282-294, April, 1932.

reports the results of an analysis of 10 early books widely used from 1896 to 1901; 10 representative textbooks in current use in 1928; and 5 recent books published since 1925, but in all cases after those represented in the second group. Lack of space prohibits inclusion of tabulations. The nature of these tabulations, however, are suggested by the following significant conclusions, which, except for the italics inserted by the present writer, are quoted:

1. Efforts on the part of writers of current and recent textbooks in plane geometry toward *establishing a concrete basis for the beginnings of demonstrative geometry* are shown in a longer and more informal introductory chapter, in the introduction of developmental exercises and material, in the provision of many concrete illustrations of geometric principles, in the use of "practical" questions and exercises, in the use of geometric instruments, and in an increased use of all types of illustrations.

2. An endeavor to meet the criticisms of the *practice of passing from one principle or theorem to another without applying these principles* in various situations is shown in the introduction of exercises in the preliminary pages, in the greatly increased emphasis given exercises immediately following theorems and problems, in the general use of the applied problem, and in the descriptions and the use of geometric instruments.

3. *Less attention to scientific rigor and strict logic* and more emphasis on a presentation from the point of view of the pupil are seen in the general acceptance without proof of many evident truths, in the use of informal proofs for many theorems formerly given formal proofs, and in the giving of informal explanations of many terms rather than precise definitions.

4. The recognition of *training in logical thinking* as a most important outcome of the study of plane geometry is evident in the decreased emphasis on the complete model proof; in the general emergence of the incomplete proof and problem and the unproved theorem and unsolved problem; in the analytic proofs and in the analyses of problems; in the provision of hints for the solution of exercises in the form of analyses of questions and the suggestion of plans of attack in theorems; in the interrogative statement of exercises; in the attempts to grade the exercises according to difficulty; in the significant increase in the number of exercises of nearly all types; in the introduction of developmental material and exercises, especially in recent textbooks; and in the grouping of theorems.

5. Attempts to effect some *correlation with other subjects* is seen in the increased numbers and percentages of algebraic exercises, especially in recent textbooks; in the introduction of trigonometric exercises and topics in many current and recent books; in the derivation and sum-

mary of formulas; in the use of algebraic analysis and notation; and in the introduction in a few books of topics from algebra, analytic geometry, and astronomy.

6. The problem of *individual differences* is given some attention in the inclusion of optional material; in differentiation in theorems, problems, and exercises; and in one book through the use of the same material for all pupils, but at different levels of attainment.

7. Particular emphasis has been given to the *improvement of the treatment of definitions, theory of limits, ratio and proportion, and loci* in many current and recent textbooks.

Miss Freeman points out that more of the authors of current and recent geometry textbooks were engaged at work in secondary schools at the time of writing than was true of the early authors. Her conclusions evidence, on the part of late writers, a greater realization of pupils' needs and better provisions for the development of independent thinking as contrasted with the memoriter type of learning so common under the older plan.

6. ADAPTATION TO PUPIL NEEDS AND INTERESTS IN PLANE GEOMETRY

Ability grouping.—In practically all the larger cities visited the classes in geometry were arranged in ability groups, based for the most part on the intelligence quotients of the pupils. The adaptation of the course of study to such groups is usually left to the individual judgment of each teacher. The feeling was expressed in some cases that such grouping is unsatisfactory and that there is more definite need of ascertaining mathematical abilities in determining the sections. In Detroit teachers of ninth-grade mathematics in the intermediate schools at the end of the year send with the pupils to senior high school a report of their estimate of their mathematical ability.

Need of proper introduction.—Teachers in charge of classes just beginning geometry sense the importance of a proper introduction to the work. While justification of the place of geometry in the curriculum is based on the unusual opportunity the subject affords for developing logical thinking rather than on its practical application, the need of the latter for developing the former is given place. At Washington High School in New York City—a 4-year high school—the

head of the department has devised prognosis tests in algebra and geometry which are given to beginning classes. The results of such tests enter into the basis on which ability groups are formed. This is one of the few schools which is shown in a previous analysis as including a unit of demonstrative geometry in the algebra course.[9] Such classes are continued the next year as fusion classes to which both plane and solid geometry are given in the tenth grade.

As shown near the opening of the present chapter, however, not many schools, even among those devoting considerable attention to intuitive geometry in the early grades, show, in the outline for plane geometry, a connection of this work with that immediately to follow. Among the schools visited, the work done at the English High School at Boston is especially noteworthy. The aim of the introductory work, which is built on the geometry of grades 7, 8, and 9, is "to enable the pupils to understand the facts of geometry and to express them in their own words, as a preliminary step to the more formal presentation to follow."

The preliminary work consists largely of constructions with ruler, compass, and protractor. Throughout these constructions opportunity is taken to build up the mathematical vocabulary of the pupil, such as, for example, the "congruence" of triangles by cutting out triangles in which three sides of the one are equal to three sides of the other. Demonstration is not begun until the pupils know the facts and understand how to apply them. Axioms are learned *as the need for them arises* in connection with the proof of different theorems.

Development of logical reasoning.—The aim of geometry shown to be expressed most often in the outlines is the development of logical reasoning. The development of independent thinking on the part of the pupil is the objective toward which the efforts of teachers in the schools visited were most consciously directed. In most of these schools this has resulted in efforts of varying nature, all of which, however, are attempts to steer the pupil clear of the "memory rut" so common to geometry. In the Fieldston Ethical

[9] For suggestive outlines of such units see Fifth Yearbook, The National Council of Teachers of Mathematics, Columbia University, New York, pp. 44-63.

Culture School and the Horace Mann School for Girls in New York, through excellent libraries and use of art materials the cultural phases of geometry are connected with its study. The laboratory plans of teaching followed at the University of Chicago High School and the John Burroughs School, Clayton, Mo., furnish opportunity for much testing and individual work with pupils. On the whole, however, the connection of mathematics with the environment is not so consciously sought in senior as in junior high schools.

Correlation.—Aside from the movement for the correlation of plane and solid geometry, there are few efforts to fuse other mathematics subjects with the teaching of plane demonstrative geometry. At the University of Chicago High School a number of geometric proofs are simplified through the use of trigonometry and algebra, and the equation is applied to geometric matter wherever possible. Many other methods are followed in this school, however, through which the pupil is taught to think analytically. Methods of proof are discussed and the pupil is taught to choose between various methods. Every opportunity is taken to bring out the relations of quantities to each other and the usefulness of the study in everyday life is impressed. Pupils work mostly at their seats, but their individual progress and abilities are tested often. Remedial work is given through practice materials.

At the Lincoln School, Teachers College, New York, an experiment is under way through which geometry and physics are to be given together over a period of two years by a representative of each department. One year of the work has already been given to a class of 29 tenth-grade pupils, and at the end of the second year it is thought possible that sufficient work may have been done to grant two units' credit in mathematics and one in physics. In general, the classes meet six days per week, four of which are devoted to mathematics and two to physics.

In the English High School, Boston, an experiment is being made in the use of a new syllabus in plane geometry which stresses measurements and the use of irrational numbers. No textbooks are used in this school, but each pupil

keeps a notebook. The teachers think that pupils are more interested and that the artificial barrier which often exists in the mind of the pupil between the study of originals and the study of theorems is obviated. The regular course of study for the tenth grade includes some algebra and trigonometry with plane geometry. Functional thinking, numerical work, and approximations are stressed throughout the course to develop common-sense notions about size relations. That interest is maintained is evidenced by the fact that in a school enrolling 2,300 pupils, 300 are this year taking twelfth-year mathematics.

While the examples given indicate correlations which are attempted, on the whole fewer efforts were discovered on the part of senior high school teachers along this line than in the junior high school field.

7. CLASSES FOR SPECIAL GROUPS

The large number of failures among pupils taking a regular college-preparatory course in geometry has led to efforts to interest pupils in courses not leading to college. The Oakland (Calif.) course of study provides for two courses for tenth-grade pupils, both of which are called "applied mathematics." One course consists of everyday problems, such as taxes, rent, sale and purchase of merchandise, and the like, but the content of the other largely parallels the regular geometry with different aims and methods. Stress is laid upon symmetry and appreciation of form structure, and the truths of geometry are arrived at largely through measurement and drawing.

Among the schools visited, the Beaver Country Day School, Brookline, Mass., postpones plane geometry until the eleventh grade and offers the tenth-grade pupil a continuation of algebra or, for those not going to college, a course in practical mathematics. In the latter course, such units as money and banking, thrift and investments, statistics and graphs, taxes, insurance, and the like are offered. At Cass Technical High School, Detroit, a course in the concepts of geometry is offered the non-college-preparatory group, the materials for which are based on a textbook written by a member of the mathematics department.

In the same city the work at the Wright Cooperative School, organized in 1928 at the request of a group of employers from large automotive plants, is especially adapted to pupils expecting to enter industry. Pupils allowed to enter this school are carefully selected and are by no means a low I. Q. group. The subject matter is constantly being changed to correspond to the practical needs and interests of pupils as revealed through careful analysis of the written work which they hand in. The aim in mathematics is to teach the pupil fundamental processes and lead him, through correlation with school shops and industry, to apply these processes intelligently. Each boy may progress at his own rate. Courses especially adapted to the following sections are offered: Manufacturing and steam engineering, commercial-automotive maintenance, and graphic arts.

8. MATHEMATICS IN GRADES 11 AND 12

As indicated in Table 13, the mathematics of grades 11 and 12 consists largely of (1) plane and solid geometry; (2) intermediate, advanced, and college algebra; and (3) trigonometry. The content and methods of teaching as outlined for these courses are for the most part of the traditional college-preparatory nature. In some centers, however, elementary phases of analytical geometry and the calculus are being taught with reported success in these grades.

At the Wadleigh High School, in New York City, the calculus has been taught for some time by John A. Swenson with regular senior high school courses. In advanced algebra the introduction of the calculus is confined almost entirely to algebraic expression, the calculus of trigonometric functions is studied in connection with the formal study of trigonometry and volumes and solids are dealt with in the study of solid geometry in the twelfth year. The work is fused with other topics as the occasion presents itself. Swenson's conclusion is that the calculus includes sufficient algebra and geometry to compensate for what ever diversion is necessary for its presentation.

In some of the schools visited the calculus or analytics, or both, are fused with the work in the upper grades, while in others they are given as separate units. One or both,

however, are included in the work of the following schools: George Washington and Wadleigh High Schools in New York City; Horace Mann School for Girls and Lincoln School, Teachers College, Columbia University, New York City; English High School and Public Latin School, Boston; Cleveland High School, St. Louis; John Burroughs School, Clayton, Mo.; and the University High School, Chicago.[10]

Some correlation is being effected in the usual branches of mathematics taught in grades 11 and 12. Joseph B. Orleans, head of the mathematics department, George Washington High School, New York, is coauthor of a recent textbook in eleventh-year mathematics which outlines the integrated course in intermediate algebra and trigonometry as taught at that school.

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[10] Additional accounts of work in the calculus may be had from the following references:

Farmer, Susie B. The Place and Teaching of Calculus in Secondary Schools. Mathematics Teacher, 20: 181–202, April, 1927.

Gugle, Maria. Revision of College-Preparatory Mathematics. Mathematics Teacher, 19: 321–328, October, 1926.

Mirick, Gordon R. Reorganization of the Tenth, Eleventh, and Twelfth Grade Course in Mathematics. Junior-Senior High School Clearing House, 5: 329–332, February, 1931.

—— and Sanford, Vera. An Elective Course in Mathematics for the Eleventh and Twelfth School Year. Mathematics Teacher, 19: 235–241, April, 1926.

Nordgaard, M. A. Introductory Calculus as a High-School Subject. Third Yearbook, The National Council of Teachers of Mathematics, Columbia University, New York, pp. 65–101.

Reeve, W. D. The Mathematics of the Senior High School. Teachers College Record, 28: 374–386, December, 1926.

——. United States. Fourth Yearbook, The National Council of Teachers of Mathematics, pp. 168–174.

Swenson, John A. Selected Topics in Calculus for the High School. Third Yearbook, The National Council of Teachers of Mathematics, pp. 109–134.

CHAPTER IV : INFLUENCES ON THE PRODUCTION AND USE OF COURSES OF STUDY

1. MACHINERY FOR PRODUCTION AND SUPERVISION

Few outlines give details concerning the manner of producing them. This whole matter, as it relates to all subject fields, is treated specifically in another section of the survey. For the most part, in the cities represented in the present study, revision is in direct charge of an assistant superintendent, who appoints a committee of usually 5 to 10 teachers to work out details of the course. Other teachers in the system usually react, however, to these details before the course is adopted by the board of education. Outside specialists are often brought in for consultation, both on general and on specific matters. The teachers usually work extra time at curriculum revision without additional pay, although in some cases substitutes are employed to do their regular work while they are engaged in curriculum construction.

The supervisory program is indicated in none of the course outlines which were examined. In all senior high schools visited, supervision is under the head of the mathematics department of that school. The same is true for almost all junior high schools, although in two or three cities the principal is the chief supervisor in such schools. In two of the cities visited, in addition to the department heads, a director is employed for general supervision of mathematics instruction in the several schools. In the other cities visited the unification of the program is in charge of an assistant or deputy superintendent.

Leadership in mathematics within the schools visited depends largely on the efforts of the individual department heads. In some schools, however, the status of the department head differs little from that of any other member of the department. Sometimes no extra time is allowed for supervision, and in such cases very little can be done. Where there is no director of mathematics for the city as a whole, leadership in mathematics must too often await many other

duties of the assistant superintendent. In some cities a mathematics club for teachers has been organized, but attendance and effort in such instances are without compulsion.

2. NEED OF TEACHER TRAINING

The necessity of training teachers in the principles on which the outline is built and on their application in the classroom is one of the strongest impressions gained from visiting schools. In systems enrolling only 500 to 600 pupils in all secondary grades, the purposes advanced by the head of the department as controlling instruction in the school seem, on the whole, reflected to a far greater degree in the work of all classroom teachers than is the case for larger schools. In one of the larger cities visited a very excellent course of study had been constructed which was sent to the teachers for criticism during the closing days of the year. As a result, few reactions were secured. The outline was nevertheless printed; it had to be shelved later because the teachers as a whole were not in sympathy with it.

Lack of cooperation was reported much more often in the senior than in the junior high school. In one of the cities employing a director of mathematics, this official considered the nature of the training of teachers in senior high school as the greatest obstacle to the successful operation of the work. Due to the fact that many of these teachers had been teaching by the same methods over a considerable period of time, they were loath to accept new ideas.

3. SELECTION AND USE OF TEXTBOOKS

As noted in Chapter I, the outlines for mathematics instruction are related to basic textbook materials in a majority of the outlines represented. In Boston the mathematics council adopts a list of textbooks, any of which the teacher may use. In the junior high school grades of Cleveland, different texts are adopted for use with the X, Y, and Z groups, representing the bright, medium, and slow pupils. In most systems and schools, however, a single textbook is adopted, which all teachers feel that they must use.

In practically all schools visited the textbook is selected through a committee appointed for that purpose, each

member of which uses his own methods in arriving at a conclusion. A school official in one of these cities felt that publishers of textbooks affected too much the choice that is made. Certainly the selection should be made on the degree to which the book chosen represents the ideas of the staff as to materials and methods of instruction rather than on any other basis.

It is for the purpose of insuring this that the methods outlined by Florence D. Fuller were followed in the selection of mathematics textbooks for the Los Angeles junior high schools.[1] Altogether, nine separate committees, with outlined procedures, analyzed different features of several textbooks, assigned certain values to each feature, and made their selection on the results revealed. Thirty per cent of value was placed upon the analysis of content; 27 per cent on presentations, discussions, methods; 15 per cent each on manner developed and on drills, tests, summaries, and reviews; the remaining 13 per cent was divided between the difficulty value of the vocabulary, illustrations, scholarship of authors, size of members, and mechanical phases.

The dependence of the classroom teacher on the textbook in use varies, of course, with different schools, but in the majority it plays a very important part. In the intermediate schools of Boston, however, textbooks representing three or four different authors are used by different members of the class. Certain classes in the senior high school use no texts. In some of the cities visited, the textbook used in one or more grades is one written by a member of the mathematics department of that city.

In St. Louis and in Detroit the new textbooks adopted for ninth-grade mathematics are being used as the basis of the course of study outline to be written later. Tests are being given by key teachers throughout these systems, and the results will be used to determine points of emphasis in high, average, and low ability groups

4. RESEARCH AND EXPERIMENT

The small number of outlines which indicate research activities in producing the course of study was indicated in

[1] Scientific Evaluation of Textbooks. Chicago, Houghton Mifflin Co., 1928.

Table 5. Some committees had no doubt made use of studies published by others or had conducted research locally in building their outlines, but such facts were not mentioned in the outlines. For the course of study at Dayton, Ohio, the mathematics teachers of grades 7 and 8 guided the pupils in keeping a record of their arithmetical experiences for one week. Some of these experiences are described in one column of the outline, while opposite them in another column are listed the habits and attitudes which are suggested by them.

Before work was begun by the production committees at Rochester a survey of the entire school system was made and the results were determinative of the materials and methods of instruction included in the new courses. Lucie L. Dower reports some suggestions on problem solving in arithmetic which are based on the results of the survey.[2]

Breslich reports some of the careful measurements carried on for many years in constructing the course for junior grades at the University of Chicago High School.[3] No topics were retained merely because of a traditional place in the curriculum. Materials were weighed through analytical tests, and much arithemetic was excluded after it was shown that pupil efficiency could be secured best through incidental reviews in practical situations.

The need of checking results is greatest while the course of study is being first applied. This is done to a considerable extent under the laboratory plans of instruction in use at the University of Chicago High School and at the John Burroughs School. It is only through such testing that diagnostic and remedial work of the proper kind can be applied to individual cases. In the latter school an especially trained employee devotes a considerable part of her time revealing needs for remedial work in the junior high school grades.

In the conduct of this investigation few cases have been revealed in which work of this nature is being done on a city-wide scale in public schools. In most city schools attempting work of this character, even where a department

[2] Problem Solving in Arithmetic. Third Yearbook, The National Council of Teachers of Mathematics, Columbia University, New York, pp. 223-267.

[3] Breslich, E. R. Reconstruction in Secondary School Mathematics. Ph. D. thesis, University of Chicago, 1926.

of research exists, the work is independent of that department. The periodic checking of results at Cleveland has already been mentioned. In this city the department of research is very intimately connected with the curriculum-revision program. In a few other cases the results of city-wide tests appear in the course-of-study outlines and are no doubt of considerable aid in helping teachers to achieve the proper standards.

On the whole, one of the most serious weaknesses revealed in this investigation is the lack of city-wide efforts in evaluating the results of instruction. In this connection the mathematics committee producing the Fifth Yearbook of the Department of Superintendence said:[4] "While many abuses have been committed in the use of tests, too much emphasis can not be placed upon the use of tests to center the attention of teachers upon the needs of pupils and the necessity of remedial measures." The types of tests available for such a program are well summarized in the report of the National Committee.[5]

In three instances State departments have given evidence of concern over this problem. The State Department of Education of Ohio has recently issued a bulletin showing the results of A State-wide Survey of the Learning and Teaching of Geometry, while a recent publication of the University of the State of New York is a monograph entitled "Courses of Study and Curriculum Offerings in Junior High Schools in New York State" in which mathematics is treated. In 1930 the State Department of Education of Maryland issued a bulletin entitled "Arithmetic Goals" which contains suggestions for testing and corrective work.

While minute care is not always given to standardized testing, some of the city school projects already described are undertaken experimentally by their directors in an effort to fit the instruction to the pupils. Examples are the courses in general mathematics and in algebra in the ninth grades at St. Louis and Detroit; the teaching of solid geom-

[4] Fifth Yearbook, Department of Superintendence, National Education Association, Washington, D. C., p. 198.

[5] The report published by Houghton Mifflin Co., 1927, pp. 133–136, contains a short summary. A more detailed account is contained in the report published in 1923.

etry with the regular course in plane geometry at Horace Mann School, Columbia University; the new course in plane geometry at English High School, Boston; and the correlation of physics and geometry at Lincoln School, Teachers College, Columbia University.

5. CORRELATION AND ARTICULATION

The analysis of topics in the outlines for grades 7, 8, and 9 has indicated a very considerable correlation of arithmetic with algebra and intuitive geometry as well as with social and economic fields in which the fundamental processes are often applied. Attempts at correlation in the senior high school grades are less frequent, but considerable effort toward the integration of plane and solid geometry and of analytics and the calculus with the work of the eleventh and twelfth grades has been noted.[6]

Scant efforts are revealed, however, of serious attempts to objectively measure the results of such integration or to articulate such work with that of the courses following. In fact, one is led sometimes to wonder whether the connection of the work of the lower secondary grades with the economic environment of the pupil has caused teachers to neglect to stress sufficiently achievement in those fundamental knowledges and skills considered necessary for all pupils.

The need of close articulation of the work of the separate school units is evident. At Oak Park, Ill., the head of the mathematics department of the 4-year high school has devised tests which are given to pupils in elementary schools from which the high school draws pupils. The analysis of these tests has enabled the elementary teachers to place greater emphasis on certain skills in which their pupils are shown to be lacking.

6. TEACHER USE OF COURSES OF STUDY

Teachers in the schools visited are allowed considerable freedom in their methods of instruction. Although there were no indications of opposition to methods imposed by higher authority, in one or two instances it was felt that

[6] The Sixth Yearbook of the National Council of Teachers of Mathematics is entirely devoted to the application of mathematics to phases of modern life.

more material for teaching was outlined than could be taught. As indicated in Table 5, Chapter I, the majority of the outlines are lacking in many phases of the teaching process about which there is need of more uniform direction.

Difficulties which teachers in the lower secondary grades seem most often to encounter concern the slow groups of pupils. Specific suggestions which may aid teachers in this connection are lacking in most outlines. In Los Angeles a separate bulletin has been issued on Directed Study in Mathematics in Junior High Schools which treats of The Study Tools, The Materials of Study, and The Will to Study. The course of study at Springfield, Mass., contains helpful suggestions on teaching pupils to read problems understandingly and on problem solving. Several of the outlines include a "vocabulary" or list of words which the pupil should understand.

At Houston, Tex., 50 pages of the junior high school outline are devoted to diagnostic and remedial practice in the four fundamentals of arithmetic. The suggestions are based on the monograph by Buswell and John.[7] Other systems use work books and practice materials, which, in several instances, are devised by teachers connected with the system.

Senior high school pupils at English High School, Boston, The John Burroughs School, and the Fieldston Ethical Culture School are encouraged in projects involving surveying, the use and construction of the transit, the slide rule, and the like. In some schools many projects which apply the mathematics learned in school are presented in the mathematics club. At the English High School members of this club make models, such as rulers with verniers, station finders, simple transits, protractors fitted with verniers sundials, and the like, which are explained at the meetings. On excursions boys determine latitudes by means of sights on the sun at noon with sextant and transit.

Several interesting activities are reported for the Newton Club at Oak Park and River Forest High School, Oak Park, Ill. Besides debates and topics presented by members, the

[7] Diagnostic Studies in Arithmetic. Supplementary Educational Monograph No. 30, The University of Chicago, Chicago, Ill.

club has had several outside speakers of note, and has exhibited films on such topics as the Einstein theory and the characteristics of an electric motor. Members of the club sometimes go in a body to lectures in Chicago and on visits to places of mathematical interest in that city.[8]

7. COLLEGE ENTRANCE REQUIREMENTS

In at least three of the cities visited the conclusion of the National Committee (p. 14), that "there appears to be no conflict of interest during this period between those pupils who ultimately go to college and those who do not," has not worked out in practice to the extent that they feel able to offer a single course in the ninth grade that will meet the demands of both groups. The college requirements in algebra have caused the ninth-grade standards in this subject to be raised beyond the abilities of a large number of pupils in this grade. Some of the teachers interviewed do not believe that a number of pupils at this age are able to comprehend algebra. As a result, committees in these cities are now revising the ninth-grade curriculum with the idea that if all pupils be required to take ninth-grade mathematics, a separate course of a much simpler nature than will meet the needs of the college-preparatory group must be provided for the slower pupils.

Some colleges have followed the recommendation of the curriculum commission preparing the mathematics report for the Fifth Yearbook of the Department of Superintendence "in favor of basing college-entrance requirements solely upon work done in the 3-year senior high school."[9] Not a sufficient number have done so, however, to lighten the responsibility felt by the school to prepare pupils for requirements in ninth-grade algebra. At the same time the larger number of pupils now finishing the ninth grade increases the number which has no chance of going to college. If pupils must take mathematics in the ninth grade, the school is under

[8] Other references to the work of the mathematics clubs are:
The Mathematics Club Meets. Mathematics Teacher, 24: 197-207, April, 1931.
Sources of program material and some types of program work which might be undertaken by high-school mathematics clubs. Mathematics Teacher, 24: 492-502, December, 1931.

[9] Fifth Yearbook, Department of Superintendence, National Education Association, Washington, D. C., p. 192.

obligation to prepare courses in accord with their needs and interests. Some schools which have tried the single course are now changing to separate courses in algebra and general mathematics, with the hope of more adequately caring for the needs of the two groups.

While the schools just considered are located in the Middle West, teachers in New England and other parts of the East appear most concerned over training for college entrance. In the senior high schools of New York City the college-entrance syllabus is followed in the upper grades of practically all schools. In many schools in these sections a definite period of time is set aside for reviews for such examinations. While the New York State Mathematics Syllabus Committee has recently issued a ninth-year syllabus which endeavors to follow the recommendations of the National Committee, the dean of the mathematics department of one of the high schools in New York City reports that straight elementary algebra is still being taught in this grade in practically all schools.[10]

The influence of college entrance on pupil election of mathematics is reflected to some degree in the comparative enrollments in elementary algebra and plane geometry, which are the usual requirement for college entrance, and in the courses following them. In New York City, for example, Orleans reports from 38.6 to 43.2 per cent of all pupils enrolled in elementary algebra and plane geometry in 1929, an enrollment of 12 per cent in intermediate algebra, and from 0.9 to 3.6 per cent enrolled in solid geometry, advanced algebra, and trigonometry.[11] Yet, of the pupils taking elementary algebra, more than 30 per cent failed during the same year.

The situation described is no doubt typical of many schools. It indicates that standards for college entrance in mathematics are of such a nature that a considerable number of pupils are unable to meet them. So long as teachers feel it necessary to enforce such standards in this grade, it will be impossible to carry out successfully the recommendation of the National Committee that ninth-grade mathematics be required of all pupils.

[10] Orleans, Joseph B. The Present Situation in Mathematics in New York City. Junior-Senior High School Clearing House, 5:384-391, February, 1931.

[11] Ibid., p. 386.

CHAPTER V : SUMMARY AND CONCLUSION

1. PURPOSE OF CHAPTER

It is not possible in this concluding chapter to present a comprehensive outline of details of instruction in the several grades that can be recommended for adoption under all conditions. What proves advantageous in one community may not prove so in another. It is possible, however, to summarize those elements that are characteristic of the majority in a selected group of schools, to call attention to lines along which further development seems to be needed, and to indicate specific practices which it is believed teachers of secondary-school mathematics would follow if in sympathy with the trends revealed.

2. GENERAL CHARACTERISTICS OF MATHEMATICS OUTLINES

Subjects treated. Often the outlines are mimeographed. While the mimeographing indicates that they are considered tentative, they are more often bound than loose leaf. In practically all cases they contain sections on objectives, teaching procedures, and materials of instruction. In both the lower and upper divisions of the secondary-school period an average of 20 to 30 per cent of the space in outlines is devoted to teaching procedures. In the lower secondary grades an average of 30 to 40 per cent of space is devoted to materials of instruction; in the upper grades this average is between 50 and 60 per cent.

Junior high school practices.—The following characteristics obtain in a majority of the outlines for grades 7, 8, and 9 which were examined:

1. *Objectives.*—Objectives are modeled considerably in accordance with the recommendations of the National Committee on the Reorganization of Mathematics in Secondary Education.

2. *Selection and organization of materials.*—Materials are organized more in relation to their appeal to children's minds than in accord with logical organization. Activities are related to children's present as well as to their future needs. Materials of local interest and from fields other than mathematics are included. Indication is given of the

relative emphasis to be placed on the different topics as well as to the amount of time to be allotted to them.

3. *Individual differences.*—Pupil activities in addition to those afforded by the textbooks are suggested.

4. *Teaching procedures.*—General procedures, as well as those in connection with specific materials, are suggested. Procedures are related to the textbook as well as to topics of local interest, and to corrective practices among pupils whose progress is irregular.

5. *Measuring the learning product.*—Suggestive tests for measuring knowledges and skills are offered.

Senior high school practices.—The outlines for grades 10, 11, and 12 in a majority of schools show considerably less detail and seem to indicate less care in their construction. Objectives are listed for each course, but not for the senior high school as a whole. They show the influence, however, of the National Committee formulation. The materials of instruction are organized logically rather than psychologically. Definite time periods are allotted to the main divisions of content. Teaching procedures are suggested which are of a general nature, but they are not connected with specific materials.

Suggestive practices.—Just what a course-of-study outline should contain may not be interpreted from this summary. There is need of investigation of the phases of instruction treated in courses of study which teachers find most helpful. It is evident that practically all are agreed that objectives, materials of instruction, and teaching procedures should receive treatment. Practices in the schools visited indicate of late that helpfulness to the teacher rather than uniformity of practice is motivating the construction of outlines. Not only such phases as individual differences, measuring the results of instruction, and references to materials from which teachers may secure help are receiving greater attention, but committees producing course outlines are concerned as well with presenting their materials in attractive form. Examples of outlines in which careful attention is given such items have been mentioned. By way of summary the following illustrations may not be out of place: Separate materials and activities for pupils on different levels of ability (Cleveland and Topeka); remedial work with pupils below the norm (Houston); correlating mathematics with other fields of in-

struction (Los Angeles); correlation with other courses in mathematics (Rochester); articulation of work with grades above and below (Pittsburgh); reference to additional materials (Los Angeles); suggestions for testing (Springfield, Mass.).

3. *MATHEMATICS IN JUNIOR HIGH SCHOOL GRADES*

Objectives.—The objectives of mathematics formulated by the National Committee seem to have influenced markedly those appearing in outlines prepared for use in grades 7, 8, and 9. The following characteristics seem to prevail:

1. Aims which relate to the practical objectives of instruction predominate.
2. They include the application of mathematics to the social and economic environment of the child.
3. Informational as well as computational mathematics is emphasized.
4. The introduction and correlation of mathematics from higher courses is stressed.
5. Disciplinary and cultural aims appear in the majority of outlines, but less often than the practical.

Although the aims reflect a leaning toward the practical side of mathematics, the outlines do not indicate a decided connection of such aims with teaching materials and procedures. There seems to be need of more careful breaking up of objectives so that they will not only connect with actual materials of instruction but that the degree to which they do connect may be measured. Mention has been made of work of this nature at Ann Arbor, Mich.

Content in grades 7 and 8.—Important characteristics of content revealed through analyses of outlines and textbooks on mathematics in grades 7 and 8 are summarized as follows:

1. Mathematics is required in almost all schools represented.
2. In more than 60 per cent of the schools the required course is designated as general mathematics or simply as "mathematics."
3. Although included more often in courses designated as general mathematics, material from intuitive geometry and algebra is often a part of courses offered as arithmetic.
4. In both arithmetic and general mathematics there is considerable application of mathematics to such economic and social uses as are afforded in the community and the business world.

5. Wide variation exists in the order of presenting topics. As between arithmetic and general mathematics, new materials rather than reviews are presented first more often in the latter, on the theory that better interest will result.

Suggestive practices.—The two problems with which teachers visited seemed much concerned were (1) proper emphasis on skill in the fundamental processes and (2) problem solving. Practices enumerated which illustrated success in the first difficulty are based for the most part on the application of the problem to a setting familiar to the child. The examples given of practices at Rochester, Cleveland, Boston, and other schools involve daily practical applications and relations suggested by thoughtful teachers. Drill is introduced successfully as incidental to other problems and is made interesting by having the pupil keep account of his own record. The following practices worked successfully in the schools mentioned earlier: Motivation through the history of mathematics, introduction of new mathematics materials into the course, correlation of mathematics with work in other fields, collection of problems by the pupils, and use of mathematics clubs.

Many difficulties arise in connection with problem solving. Additional studies are needed which will ascertain the nature of pupil responses to different types of problems and to determine the teaching procedures through which best results are obtained. Attention has been called to practices in certain of the private or laboratory schools in which the teacher uses every opportunity to gain knowledge of the abilities, achievements, and interests of each child which she may skillfully make use of to train the child in quantitative thinking. Plans mentioned which teachers believed were successful include the collection of the problems by the pupils at Cleveland, the teaching of problems by types at the John Burroughs School, the selection of problems of interest to the pupil at Dayton, and the teaching of analysis and reading of problems with understanding at Springfield, Mass.

Mathematics in grade 9.—Algebra and general mathematics appear to be about equally represented in the offering of

grade 9. They are required in about half of the schools whose programs were analyzed. For each of these courses there is indicated a departure from traditional content, but not as great as appeared for mathematics in grades 7 and 8. In general the analysis of topics in algebra and general mathematics which were reported reveal:

1. In general mathematics there is less emphasis on the fundamental operations, factors, and fractions, but more emphasis on the meaning and use of the formula.
2. Textbook analyses show a much greater inclusion of intuitive geometry in the general mathematics for grade 9.
3. Trends in textbooks in elementary algebra indicate a limitation in late books on drill in algebraic technique.

Schools which require mathematics in grade 9 are having difficulty in adapting materials of instruction to the needs and interests of all pupils of these grades. There is need of more careful study of the mathematical content adapted to pupils who take their final work in this field in grade 9. The results at Cleveland, Detroit, and St. Louis, where efforts are now being made to assemble materials which will meet the needs of the pupil not going to college, will be of interest in this connection. Schools in which mathematics is not required in this grade, as at Boston and in the private or laboratory schools, appear to be attacking the same problem with some success through much emphasis in earlier grades on the relation of quantities and through individual help to pupils whose weaknesses are revealed through careful testing.

4. *MATHEMATICS IN SENIOR HIGH SCHOOL GRADES*

The objectives.—As contrasted with junior high school grades, the aims tabulated from outlines for mathematics in grades 10, 11, and 12 reveal more emphasis on disciplinary and cultural values than on the practical outcomes of instruction. Such aims are not definitely connected with the materials of instruction, however, and they appear in most outlines, as of the traditional, college-preparatory nature.

The total offering.—On the whole, not a large number of the schools represented are providing courses adapted to pupils not going to college. It is of significance that only 13 of the

centers represented offer courses in vocational mathematics. There is need of a careful study of what are the methematical needs of pupils enrolled in business and industrial courses that are not met through general mathematics courses in junior high school. The efforts of the department at Wright Cooperative School, Detroit, to provide such materials are worthy of note. It is significant also that Detroit and Oakland are offering courses in geometry concepts for pupils not going to college.

The offering in grade 10.—While geometry is the course which is offered most often in grade 10, enough variations from this practice are found to indicate that the matter is not entirely settled. College-entrance requirements are affecting such practices considerably. Some schools are attempting to introduce algebra in sufficient quantities in the geometry course so that algebra can again be pursued in grade 11 with little loss of time. Others are following the European practice, in which, during the same year, geometry is taught part of the week and algebra the remainder. The success of these arrangements at the University of Chicago High School and the Public Latin School, Boston, may be instrumental in causing a greater number to change from the usual.

There is evidence that teachers feel the need of more careful diagnosis of abilities of pupils before they are allowed to enroll in demonstrative geometry. The introduction of intuitive geometry in junior high school makes necessary a more adequate evaluation of the contribution of such work to the course in demonstrative geometry.

Suggestive practices.—On the whole, less widespread efforts to connect mathematics with the practical needs of pupils in senior than in junior high school are evidenced in this study. Work in grades 11 and 12 is still almost entirely college preparatory. Results obtained in schools fusing analytical geometry and the calculus with advanced algebra, solid geometry, and trigonometry are worthy of more notice. Attempts in a few centers to bring about closer integration of plane and solid geometry should become more widespread. Schools report that they have made the tenth-grade course

more interesting through teaching intuitively certain elements of solid geometry. No school has yet reported a correlation, however, which will enable the acquirement of the entire needs of these two courses in one year.

The changes which have been indicated in the content of current and recent textbooks, as well as results in some of the cities visited, should encourage more widespread efforts to bring about logical reasoning rather than memorization on the part of the pupil. Certain of the practices enumerated in the text, such as a more careful introduction to plane geometry, the elimination of many theorems having for the most part only traditional value, emphasis on methods of proof of original problems, and emphasis on the function concept in teaching, appear worthy of more general application.

5. INFLUENCES ON THE PRODUCTION AND USE OF COURSES OF STUDY

Production of outlines.—Although the actual construction of outlines is usually in the hands of small committees of teachers, in most of the cities visited all teachers, either through assignments incident to production or through training in use of the tentative outline, were made familiar with the contents of the new course of study. School officials seemed convinced of the value of such participation as a means of improving teachers in service, and for this reason strive for 100 per cent participation. Greater cooperation seems to be secured in junior than in senior high school.

Extensive efforts toward correlating the work of the separate grades or divisions of the school or toward research and the use of objective studies are not indicated. The lack of a city-wide program for measuring the results of instruction seems one of the greatest weaknesses in mathematics instruction. The reader is reminded of the careful work of this nature being done at Cleveland, the John Burroughs School, and at the University of Chicago High School.

Use of outlines.—In most cases outlines are connected with an adopted textbook. In Boston, teachers appeared independent of any one textbook. Improved methods in the selection of textbooks are indicated in the method described as in use at Los Angeles. Examples of out-of-school activities, such

as class visits and programs of mathematics clubs, will be recalled. Likewise, worthy of mention are such supplementary bulletins as the one at Los Angeles on how to study and the attention devoted to diagnostic and remedial work at Houston. Especially in the East, teachers in grades 9 to 12 seem influenced considerably by college-entrance examinations. Attention has been called, however, to efforts in certain cities to prepare courses for other than college-preparatory pupils.